MATH 20100 section 1.2

2) $f(x)=\sqrt{x+1}$ $g(x)=\sqrt{x-1}$

domain: $x+1\geq 0$
$x\geq -1$
$-1\leq x$
$[-1,\infty)$

domain: $x-1\geq 0$
$x\geq 1$
$1\leq x$
$[1,\infty)$

$(f+g)(x)=f(x)+g(x)=(\sqrt{x+1})+(\sqrt{x-1})=\sqrt{x+1}+\sqrt{x-1}$

the domain of $(f+g)(x)$ is the intersection of domains computed above. $\underline{\underline{[1,\infty)}}$

$(f\cdot g)(x)=f(x)\,g(x)=(\sqrt{x+1})(\sqrt{x-1})=\sqrt{(x+1)(x-1)}=\sqrt{x^2-1}$

$x^2-1\geq 0$ yealds $(-\infty,-1]\cup[1,\infty)$ but just like above we need to take the intersection with the domain of $g(x)$; thus, domain is $[1,\infty)$

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4) $f(x)=1$ domain: $(-\infty,\infty)$ $g(x)=1+\sqrt{x}$ domain: $x\geq 0$, $0\leq x$ $[0,\infty)$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{1}{1+\sqrt{x}}$ domain: $x>0$, $0<x$, $(0,\infty)$ and taking intersection with the domain of $g(x)$ [because $f(x)$ is $(-\infty,\infty)$]

the domain is $\underline{\underline{(0,\infty)}}$

$\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}=\frac{1+\sqrt{x}}{1}=1+\sqrt{x}$ since $f(x)$ is good for $(-\infty,\infty)$

the domain is $\underline{\underline{[0,\infty)}}$

6) $f(x) = x - 1 \qquad g(x) = \dfrac{1}{x+1}$

a) $f(g(\tfrac{1}{2})) = f\left(\dfrac{1}{(\frac{1}{2})+1}\right) = f\left(\dfrac{1}{\frac{3}{2}}\right) = f\left(\dfrac{2}{3}\right) = \left(\dfrac{2}{3}\right) - 1 = \underline{\underline{\dfrac{-1}{3}}}$

b) $g(f(\tfrac{1}{2})) = g((\tfrac{1}{2}) - 1) = g(\tfrac{-1}{2}) = \dfrac{1}{(\frac{-1}{2})+1} = \dfrac{1}{\frac{1}{2}} = \underline{\underline{2}}$

c) $f(g(x)) = f\left(\dfrac{1}{x+1}\right) = \left(\dfrac{1}{x+1}\right) - 1 = \dfrac{1}{x+1} - 1\left(\dfrac{x+1}{x+1}\right) = \dfrac{1-(x+1)}{x+1} = \underline{\underline{\dfrac{-x}{x+1}}}$

d) $g(f(x)) = g(x-1) = \dfrac{1}{(x-1)+1} = \underline{\underline{\dfrac{1}{x}}}$

e) $f(f(2)) = f((2)-1) = f(1) = (1) - 1 = \underline{\underline{0}}$

f) $g(g(2)) = g\left(\dfrac{1}{(2)+1}\right) = g\left(\dfrac{1}{3}\right) = \dfrac{1}{(\frac{1}{3})+1} = \dfrac{1}{\frac{4}{3}} = \underline{\underline{\dfrac{3}{4}}}$

g) $f(f(x)) = f(x-1) = (x-1) - 1 = \underline{\underline{x - 2}}$

h) $g(g(x)) = g\left(\dfrac{1}{x+1}\right) = \dfrac{1}{(\frac{1}{x+1})+1} = \left(\dfrac{\frac{1}{1}}{\frac{1}{x+1} + \frac{1}{1}}\right)\left(\dfrac{\frac{x+1}{1}}{\frac{x+1}{1}}\right) = \dfrac{x+1}{1+(x+1)} = \underline{\underline{\dfrac{x+1}{x+2}}}$

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8) $f(x) = 3x + 4 \qquad g(x) = 2x - 1 \qquad h(x) = x^2$

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2) - 1) = f(2x^2 - 1)$

$= 3(2x^2 - 1) + 4 = 6x^2 - 3 + 4 = \underline{\underline{6x^2 + 1}}$

10) $f(x) = \frac{x+2}{3-x}$ $\quad g(x) = \frac{x^2}{x^2+1}$ $\quad h(x) = \sqrt{2-x}$

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{2-x})) = f\left(\frac{(\sqrt{2-x})^2}{(\sqrt{2-x})^2+1}\right)$$

$$= f\left(\frac{2-x}{(2-x)+1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\left(\frac{2-x}{3-x}\right)+2}{3-\left(\frac{2-x}{3-x}\right)}$$

$$= \left(\frac{\frac{2-x}{3-x}+\frac{2}{1}}{\frac{3}{1}-\frac{2-x}{3-x}}\right)\left(\frac{\frac{3-x}{1}}{\frac{3-x}{1}}\right) = \frac{(2-x)+2(3-x)}{3(3-x)-(2-x)} = \frac{2-x+6-2x}{9-3x-2+x} = \underline{\underline{\frac{8-3x}{7-2x}}}$$

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12) $f(x) = x-3$ $\quad g(x) = \sqrt{x}$ $\quad h(x) = x^3$ $\quad j(x) = 2x$

a) $y = 2x-3 = f(j(x)) = (f \circ j)(x)$

b) $y = x^{\frac{3}{2}} = (\sqrt{x})^3 = h(g(x)) = (h \circ g)(x)$

or

$y = x^{\frac{3}{2}} = \sqrt{x^3} = g(h(x)) = (g \circ h)(x)$

c) $y = x^9 = (x^3)^3 = h(h(x)) = (h \circ h)(x)$

d) $y = x-6 = x-3-3 = (x-3)-3 = f(f(x)) = (f \circ f)(x)$

e) $y = 2\sqrt{x-3} = j(g(f(x))) = (j \circ g \circ f)(x)$

f) $y = \sqrt{x^3-3} = g(f(h(x))) = (g \circ f \circ h)(x)$

14) | | $g(x)$ | $f(x)$ | $(f \circ g)(x)$ |
|---|---|---|---|
| a) | $\frac{1}{x-1}$ | $\lvert x \rvert$ | $\left\lvert \frac{1}{x-1} \right\rvert$ |
| b) | $x+1$ | $\frac{x-1}{x}$ | $\frac{x}{x+1}$ |
| c) | x^2 | $\sqrt{x}$ | $\lvert x \rvert$ |
| d) | $\sqrt{x}$ | x^2 | $\lvert x \rvert$ |

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16) $f(x) = 2 - x$ $\quad g(x) = \begin{cases} -x & -2 \le x < 0 \\ x-1 & 0 \le x \le 2 \end{cases}$

a) $f(g(0)) = f(-1) = 2-(-1) = \underline{\underline{3}}$

b) $g(f(3)) = g(2-(3)) = g(-1) = -(-1) = \underline{\underline{1}}$

c) $g(g(-1)) = g(-(-1)) = g(1)$
$= (1)-1 = \underline{\underline{0}}$

d) $f(f(2)) = f(2-(2)) = f(0) = 2-(0)$
$= \underline{\underline{2}}$

e) $g(f(0)) = g(2-(0)) = g(2)$
$= (2)-1 = \underline{\underline{1}}$

f) $f(g(\frac{1}{2})) = f((\frac{1}{2})-1) = f(-\frac{1}{2})$
$= 2-(-\frac{1}{2}) = \underline{\underline{\frac{5}{2}}}$

18) $f(x) = x^2 \qquad g(x) = 1 - \sqrt{x}$

a) $(f \circ g)(x) = f(g(x)) = f(1-\sqrt{x}) = (1-\sqrt{x})^2 = 1 - 2\sqrt{x} + x$

b) domain: $x \geq 0 \Rightarrow 0 \leq x \quad [0, \infty)$

c) range: $[0, \infty)$ {evaluate some numerical values to confirm}

a) $(g \circ f)(x) = g(f(x)) = g(x^2) = 1 - \sqrt{x^2} = 1 - |x|$

b) domain: $(-\infty, \infty)$ c) range: $(-\infty, 1]$

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20) $f(x) = 2x^3 - 4 \qquad (f \circ g)(x) = x + 2$

$x + 2 = f(g(x)) = 2(g(x))^3 - 4$

$x + 6 = 2(g(x))^3$

$\frac{x+6}{2} = (g(x))^3$

$\sqrt[3]{\frac{x+6}{2}} = g(x)$

let $g(x) = \sqrt[3]{\frac{x+6}{2}} = \sqrt[3]{\frac{1}{2}x + 3}$

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24) blue $y = x^2$

red is 3 units up so $y = x^2 + 3$

yellow is 5 units down so $y = x^2 - 5$

26) blue vertex: $(1,4)$ $y=-(x-(1))^2+(4)=\underline{\underline{-(x-1)^2+4}}$

red vertex: $(-2,3)$ $y=-(x-(-2))^2+(3)=\underline{\underline{-(x+2)^2+3}}$

yellow vertex: $(-4,-1)$ $y=-(x-(-4))^2+(-1)=\underline{\underline{-(x+4)^2-1}}$

green vertex: $(2,0)$ $y=-(x-(2))^2+(0)=\underline{\underline{-(x-2)^2}}$

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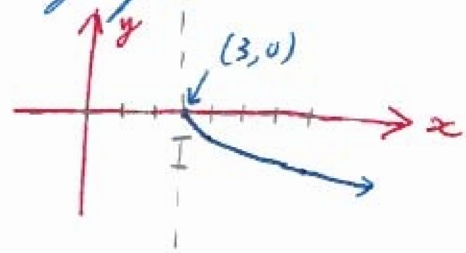
for 32, 34, 36 I'll only draw the shifted graph

32) $\underbrace{y=-\sqrt{x} \quad \text{Right } 3}$

$y=-\sqrt{(x-(3))}=-\sqrt{x-3}$



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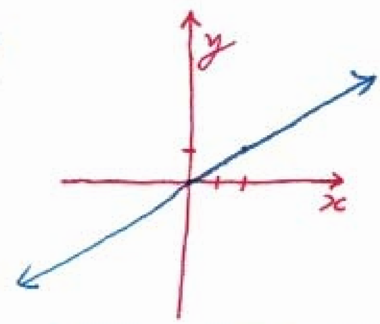
34) $\underbrace{y=\frac{1}{2}(x+1)+5 \quad \text{Down } 5, \text{ right } 1}$

$y=\frac{1}{2}\left((x-(1))+1\right)+5+(-5)$

$y=\frac{1}{2}x+0=\frac{1}{2}x$



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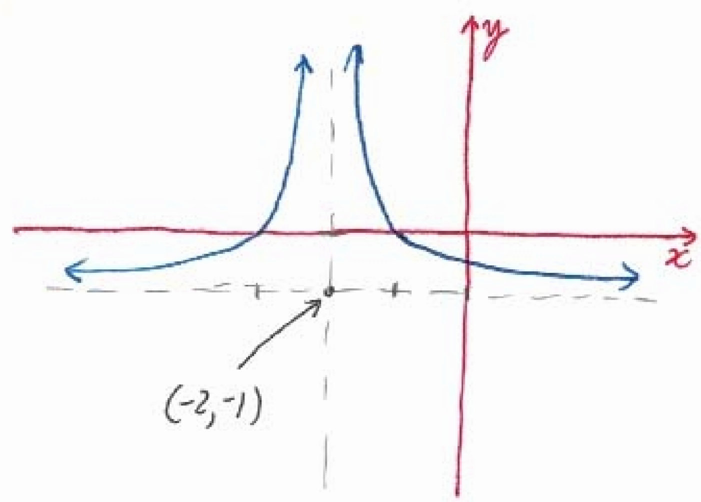
36) $\underbrace{y=\frac{1}{x^2} \quad \text{Left } 2, \text{ down } 1}$

$y=\frac{1}{(x-(-2))^2}+(-1)$

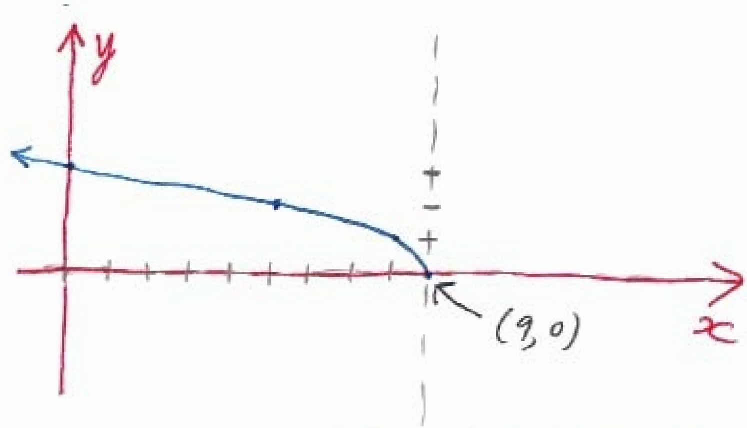
$y=\frac{1}{(x+2)^2}-1$



38) $y = \sqrt{9-x} = \sqrt{-x+9} = \sqrt{-(x-9)} = \sqrt{-(x-(9))} + (0)$

"vertex": $(9,0)$

[like $y = \sqrt{-x}$]



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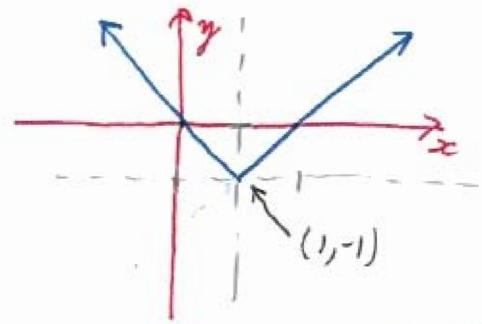
40) $y = |1-x| - 1 = |-x+1| - 1 = |-(x-1)| - 1 = |-(x-(1))| + (-1)$

"center": $(1,-1)$

[like $y = |-x|$]



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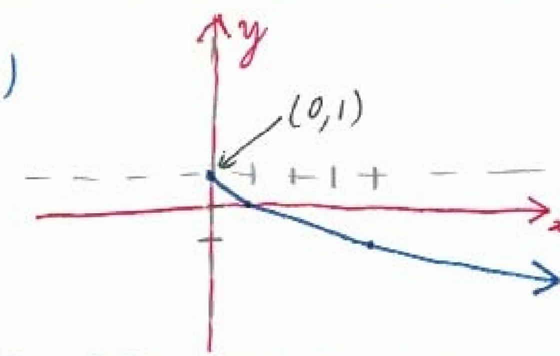
42) $y = 1 - \sqrt{x} = -\sqrt{(x-(0))} + (1)$

"vertex": $(0,1)$

[like $y = -\sqrt{x}$]



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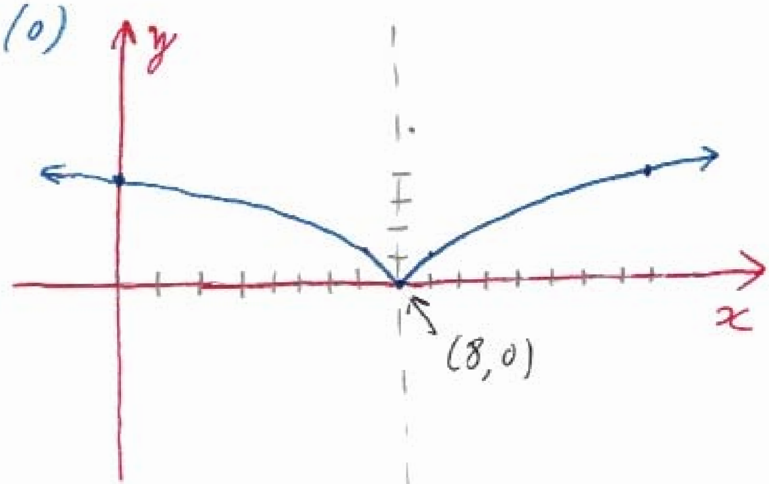
44) $y = (x-8)^{2/3} = (x-(8))^{\frac{2}{3}} + (0)$

"center": $(8,0)$

[like: $y = x^{\frac{2}{3}}$]



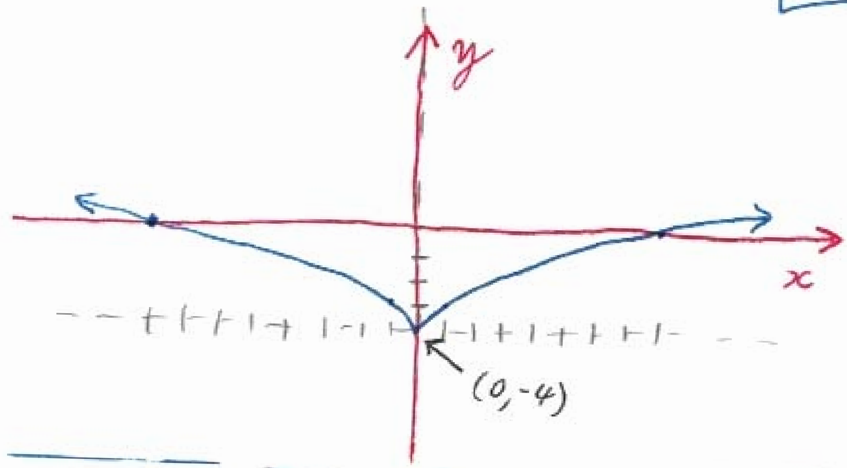
46) $y + 4 = x^{\frac{2}{3}}$

$y = x^{\frac{2}{3}} - 4$

$y = (x-(0))^{\frac{2}{3}} + (-4)$

"center": $(0,-4)$

[like $y = x^{\frac{2}{3}}$]

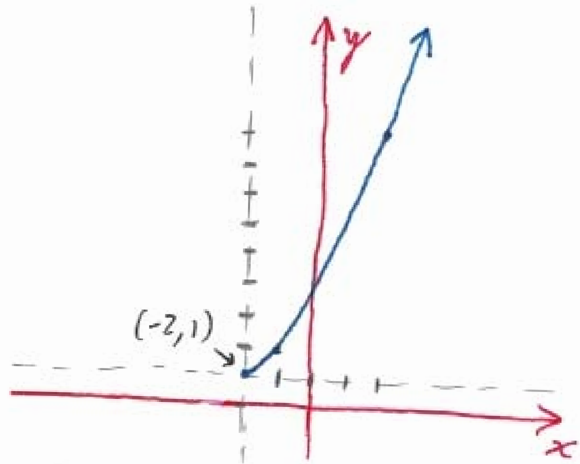


48) $y = (x+2)^{\frac{3}{2}} + 1$

$y = (x-(-2))^{\frac{3}{2}} + (1)$

"center": $(-2,1)$

[like $y = x^{\frac{3}{2}}$]

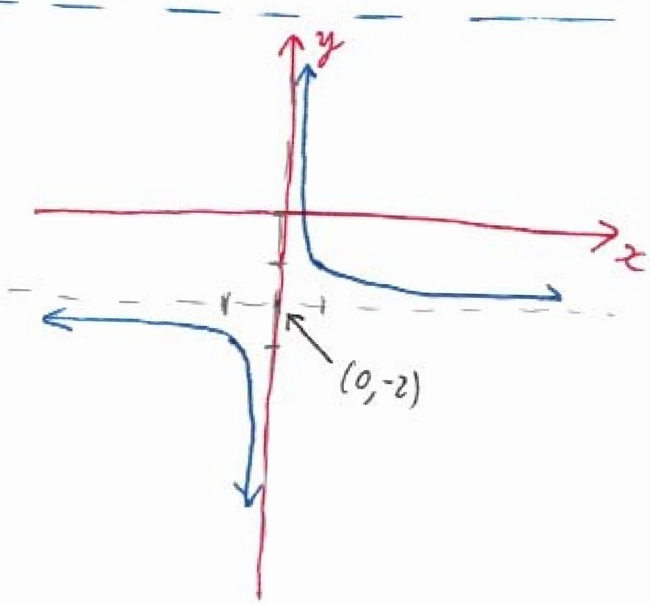


50) $y = \frac{1}{x} - 2$

$y = \frac{1}{(x-(0))} + (-2)$

"center": $(0,-2)$

[like $y = \frac{1}{x}$]

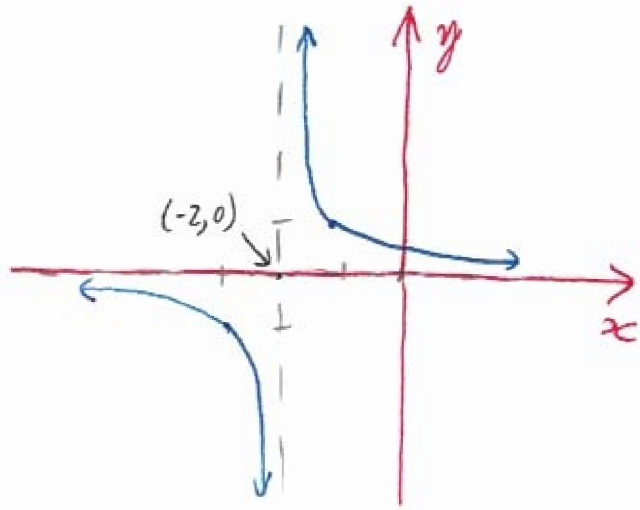


52) $y = \frac{1}{x+2}$

$y = \frac{1}{x-(-2)} + (0)$

"center": $(-2,0)$

[like $y = \frac{1}{x}$]

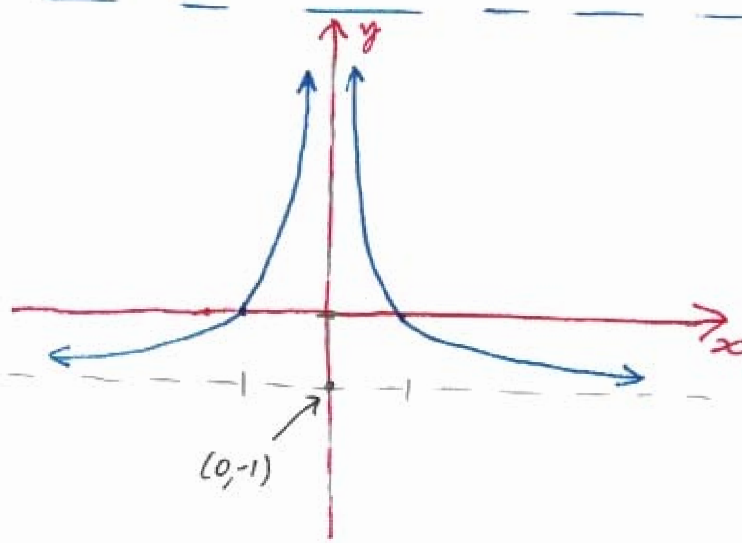


54) $y = \frac{1}{x^2} - 1$

$y = \frac{1}{(x-(0))^2} + (-1)$

"center": $(0,-1)$

[like $y = \frac{1}{x^2}$]

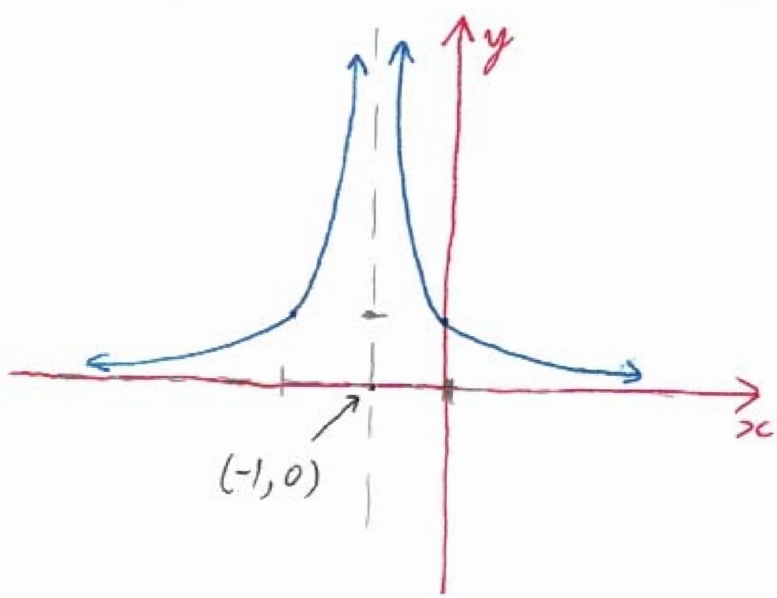


56) $y = \frac{1}{(x+1)^2}$

$y = \frac{1}{(x-(-1))^2} + (0)$

"center": $(-1,0)$

[like $y = \frac{1}{x^2}$]



60) $y = x^2 - 1$ compressed horizontally by a factor of 2

[replace x by $2x$]

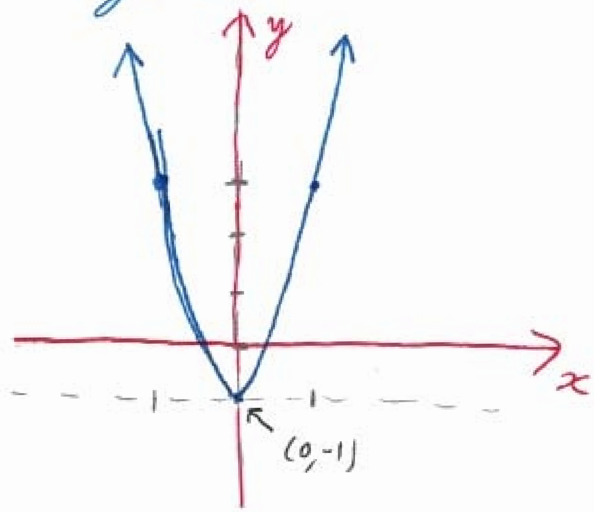
$y = (2x)^2 - 1$

$y = 4x^2 - 1$

$y = 4(x-(0))^2 + (-1)$

vertex: $(0,-1)$

like [$y = 4x^2$]



Only the modified graph drawn for 60 & 62

62) $y = 1 + \frac{1}{x^2}$ stretched horizontally by a factor of 3

$y = 1 + \frac{1}{(\frac{1}{3}x)^2}$ [replace x by $\frac{1}{3}x$]

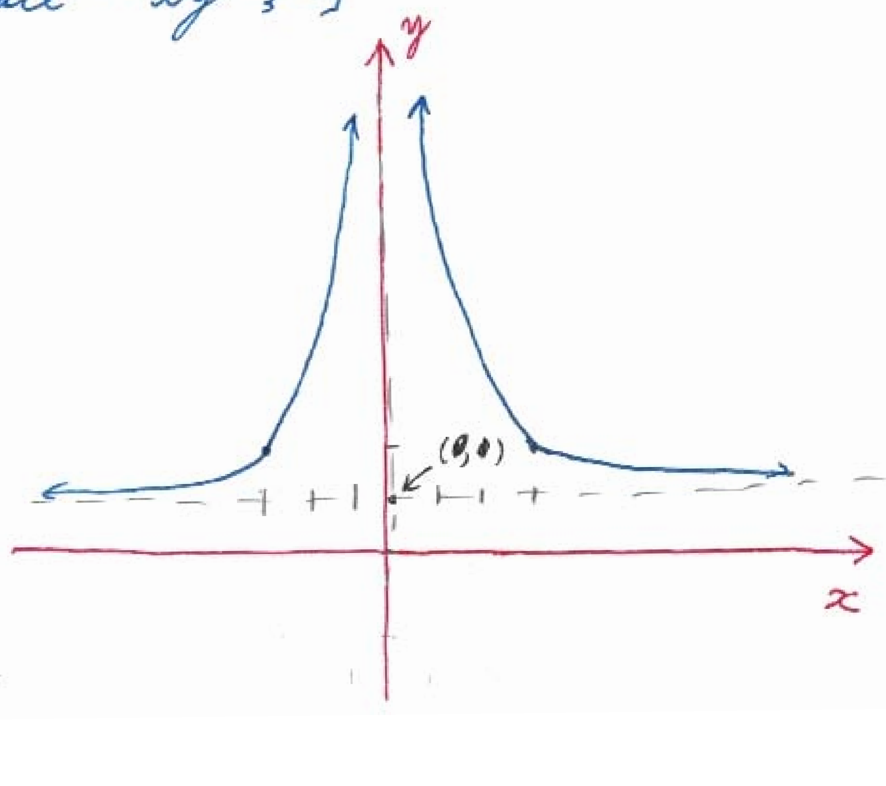
$y = 1 + \frac{1}{\frac{1}{9}x^2}$

$y = 1 + \frac{9}{x^2}$

$y = \frac{9}{(x-(0))^2} + (1)$

"center": $(0,1)$

[like $y = \frac{9}{x^2}$]



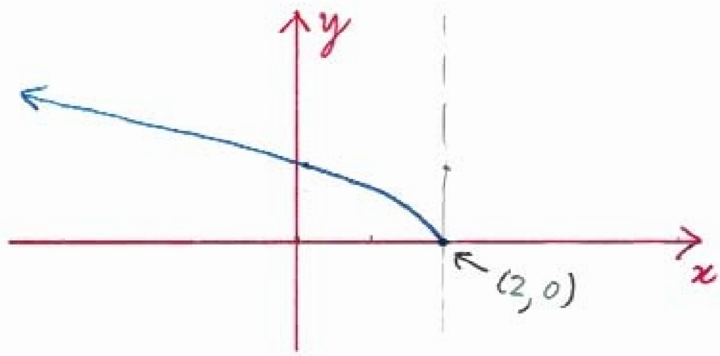
70) $y = \sqrt{1 - \frac{x}{2}} = \sqrt{\frac{-x}{2} + 1} = \sqrt{\frac{-x}{2} + \frac{2}{2}} = \sqrt{\frac{1}{2}(-x+2)}$

$= \sqrt{\frac{1}{2}}\sqrt{-x+2} = \frac{1}{\sqrt{2}}\sqrt{-(x-2)} = \frac{1}{\sqrt{2}}\sqrt{-(x-(2))}$

H-shift: +2
right 2 units

[like: $y = \sqrt{-x}$]   "vertex": (2, 0)
center

stretched vertically
by factor of $\frac{1}{\sqrt{2}}$
or
compressed vertically
by factor of $\sqrt{2}$



72) $y = (1-x)^3 + 2 = (-x+1)^3 + 2 = (-(x-1))^3 + 2 = -(x-(1))^3 + (2)$

H-shift: +1
Right 1 unit

V-shift: +2
up 2 units

"center": (1, 2)

[like $y = -x^3$]