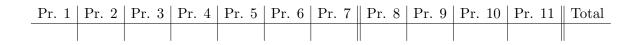
# THE CITY COLLEGE OF NEW YORK DEPARTMENT OF MATHEMATICS SPRING 2016 MATH 392, FINAL EXAMINATION

YOUR NAME (print and sign):

### NAME OF YOUR INSTRUCTOR:



#### **INSTRUCTIONS:**

- There are a total of 11 problems.
- DO ALL PROBLEMS 1 THROUGH 7 AND THREE OF THE FOUR PROBLEMS 8-11. IN THE TABLE ABOVE, CROSS OUT ONE PROB-LEM AMONG PROBLEMS 8-11 THAT YOU OMITTED.
- Each problem is worth 10 points.
- Notes, books and calculators are not to be used.
- All work on this exam is to be your own.
- Read each problem carefully. Be sure to show your work. Remember that it is your obligation to answer each question clearly and in a way that convinces the grader that you understand how to solve the problem.
- Stop working immediately at the end of the exam when time is called.

1. Find the arc-length parametrization of the curve that is the intersection of the elliptic cylinder  $x^2 + \frac{y^2}{2} = 1$  and the plane z = x - 2. Use s as the arc-length parameter with s = 0 corresponds to the point (1, 0, -1). Specify the limits for s.

**2.** Find the potential of the vector field  $\vec{F}(x, y, z) = e^{xy/z} \left( (1 + \frac{xy}{z})\vec{i} + \frac{x^2}{z}\vec{j} - \frac{x^2y}{z^2}\vec{k} \right)$ , and use the Fundamental Theorem for Line Integrals to evaluate

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the straight line segment from (1, 1, 1) to (2, 2, 1). Can one choose an arbitrary piece-wise smooth curve C with initial point (1, 1, 1) and terminal point (2, 2, 1) instead of the straight line segment?

**3.** Determine whether the following system of linear equations is consistent, and, if it is, find the set of solutions.

$$\begin{cases} 3x_1 + 4x_2 - x_4 = -5 \\ -x_2 - 6x_3 + 13x_4 = -4 \\ 2x_3 + x_4 = 3 \end{cases}$$

### 4. Use Green's theorem to find

$$\int_C xy^2 \, dx + x^3 y^2 \, dy,$$

 $\int_C \int_C \int_C y \, dy,$  where C is the boundary of the region  $D = \{(x, y) : |x| + |y| \le 1, x \ge 0\}$ , oriented positively (i.e., counterclockwise).

5. Evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S},$$

where  $\vec{F}(x,y,z) = x\vec{i}$ , and S is the part of the paraboloid  $z = -x^2 - y^2 + 1$  above the xy-plane, oriented upward.

6. Use the Divergence Theorem to evaluate the integral

$$\iint_{S} \vec{F} \cdot d\vec{S},$$

where  $\vec{F}(x,y,z) = x^2 y z \vec{k}$ , and S is the positively oriented (i.e., outward pointing normal) boundary of the cube

$$E = \{(x, y, z) \colon 0 \le x \le 1, \ 0 \le y \le 1, \ |z| \le 1/2\}.$$

7. Solve the following linear system of ODE's.

$$\begin{cases} y_1' = 5y_1 - 2y_2\\ y_2' = -2y_1 + y_2 \end{cases}$$

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## 8. Evaluate the triple integral

$$\iiint_E z \, dV,$$

where E is the region in the first octant, contained in  $z^2 \ge x^2 + y^2$ , and bounded by the sphere  $x^2 + y^2 + z^2 = 4$  and the plane z = 1.

9. Evaluate the surface integral

$$\iint_{S} (2x - y + z) dS,$$

 $\int \int_{S} (-x - y + z) dx$ , where S is the surface that is the intersection of the plane x + 2y + 2z = 1 with the first octant.

10. Use Stokes' Theorem to find the line integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where  $\vec{F}(x, y, z) = y\vec{i} + z\vec{j} + 2x\vec{k}$ , and *C* is the circle in the plane x + z = 1, centered at the point (1, 0, 0), whose radius is 2, and that is oriented clockwise when viewed from the origin.

**11.** Find the inverse to the following matrix

$$A = \begin{bmatrix} 1 & -2 & 6 \\ -4 & 9 & -23 \\ -1 & 2 & -5 \end{bmatrix}.$$

Use  $A^{-1}$  to solve the following system of linear equations

$$\begin{cases} x_1 - 2x_2 + 6x_3 = 2\\ -4x_1 + 9x_2 - 23x_3 = 0\\ -x_1 + 2x_2 - 5x_3 = -1 \end{cases}$$

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