Division of Science | The City College of New York | Department of Mathematics

MATH195 – Precalculus

Sample Final Exam A

First Name:	Last Name:
EMPLID:	

Directions:

- **NO** notes, calculators, or other electronic devices allowed. ***All electronic devices must be turned off and placed out of sight or they will be confiscated for the duration of the exam.***
- Read each problem carefully. Unless otherwise instructed, be sure to show your work.
- Remember that it is your ***responsibility*** to answer each question clearly and in a way that convinces the grader that you understand how to solve each problem.

- GOOD LUCK!

You can use this page as scrap. However any work done on this page will not be graded.

Answer all 21 questions. ***You must show all of your work*** as neatly and clearly as possible and indicate the final answer in the box for each **non-graph** question. For all **graph** questions, you should sketch your graph on the grid provided.

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1. (8 points) Let $P(x) = x^3 + 2x^2 - 9x - 18$.

   (a) (4 points) Use the Factor Theorem to show that $x + 2$ is a factor of $P(x)$.
   Show your work in the box below:

   (b) (4 points) Using the result from part (a), factor $P(x)$ completely.

Write your answer in the box below:

2. (8 points) The grey squirrel population in a certain region has a relative (continuous)growth rate of 8 percent per year. It is estimated that the population in the year 2013 was 18,000.

   (a) (4 points) Find a function that models the population t years after 2013 ($t = 0$ for 2013).

Write your answer in the box below:

   (b) (4 points) After how many years will the grey squirrel population reach 25,000?

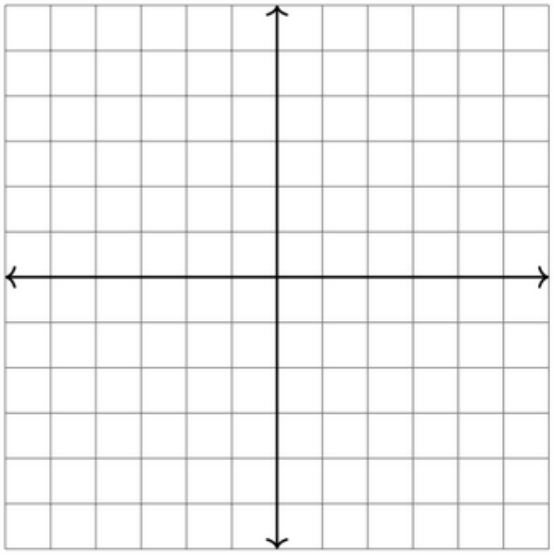
Write your answer in the box below:

3. (8 points) Consider the equation of the ellipse $x^2 + 4x + 4y^2 - 8y + 4 = 0$.

(a) (4 points) Write the equation in standard form.

Write your answer in the box below:

(b) (4 points) Graph the ellipse on the axes below.



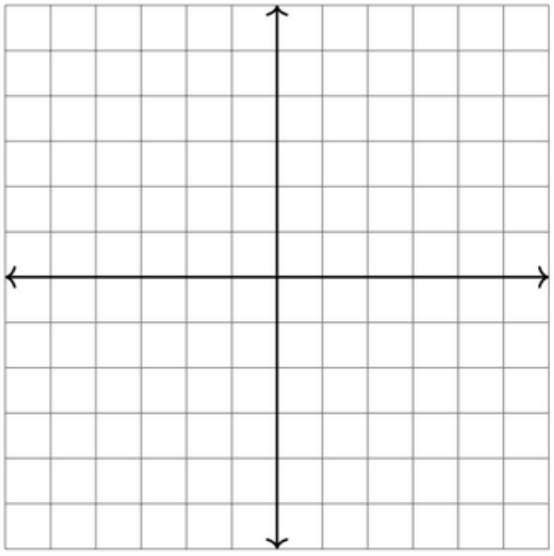
4. (8 points) Consider the system of nonlinear equations.

$$\begin{cases} x^2 + y^2 = 1 \\ y - x = 1 \end{cases}$$

(a) (4 points) Solve the system of equations. Write your answers in coordinate point form.

Write your answer in the box below:

(b) (4 points) Graph the system on the axes below. Clearly label the solutions to the system.



5. (4 points) Evaluate and simplify $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$. Express your answer in radians.

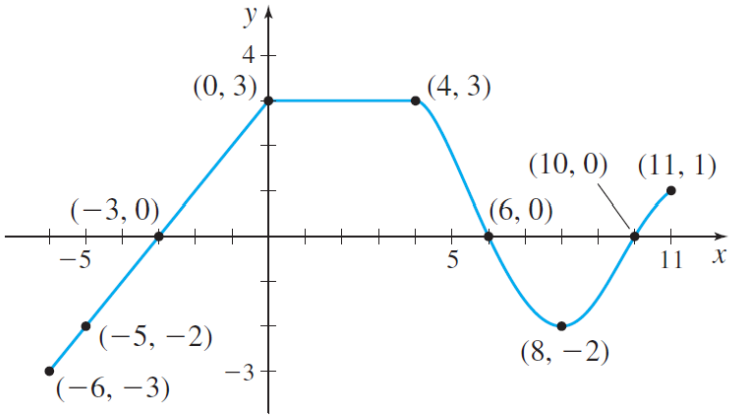
Write your answer in the box below:

6. (4 points) Evaluate the difference quotient $\frac{f(1+h)-f(1)}{h}$ when $f(x) = x^2 + 2x$.

Write your answer in the box below:

$\frac{f(1+h)-f(1)}{h} =$

7. (4 points) Use the graph of the function f below to find the average rate of change of f between $x = 0$ and $x = 4$.



Write your answer in the box below:

8. (4 points) Find $\sin 2\theta + \cos\theta$ if $\tan\theta = \dfrac{3}{4}$ and θ is in Quadrant III.

Write your answer in the box below:

$\sin 2\theta + \cos\theta =$

9. (4 points) Solve the trigonometric equation $4\sin\theta + 3\sqrt{3} = \sqrt{3}$, for all values of θ on the interval $0 \leq \theta \leq 2\pi$.

Write your answer in the box below:

$\theta =$

10. (4 points) Use the table below to solve the equation $f(x) = \dfrac{\sqrt{3}}{2}$

X	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
f(x)	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0

Write your answer in the box below:

$x =$

11. (4 points) Find the radian angle θ when the radius r of the circle is 10 and the length s of the circular arc is 25.

Write your answer in the box below:

$\theta =$

12. (4 points) Evaluate $\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$.

Write your answer in the box below:

$\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) =$

13. (4 points) Find the distance between the points $(-2, 5)$ and $(10, 0)$.

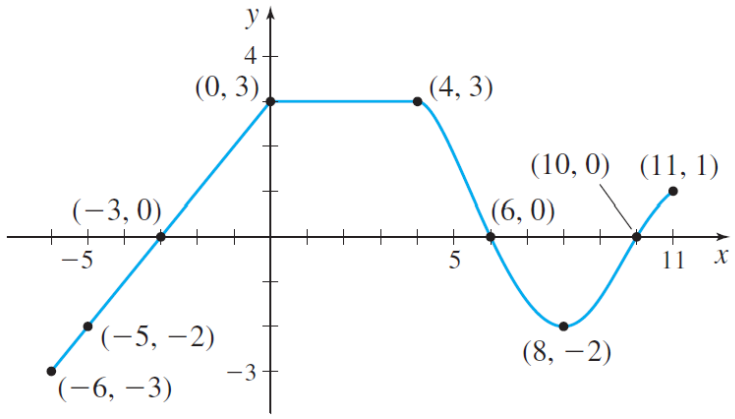
Write your answer in the box below:

14. (4 points) Solve the exponential equation $2(5 + 3^{x+1}) = 100$ for x. Express your answer in terms of logarithms.

Write your answer in the box below:

$x =$

15. (4 points) The graph of $y = f(x)$ is given below. Use the graph to find the domain of f. Express your answer using interval notation.



Write your answer in the box below:

16. (4 points) Let $f(x) = \begin{cases} x - 1 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$ and let $g(x) = \ln(x + 2)$ for $x > -2$. Find $f(g(-1))$.

Write your answer in the box below:

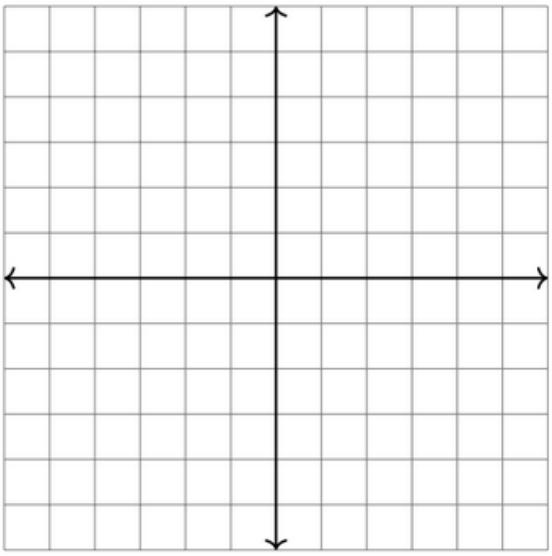
$f(g(-1)) =$

17. (4 points) Let $f(x) = \dfrac{4}{4-x}$. Find $f^{-1}(2)$.

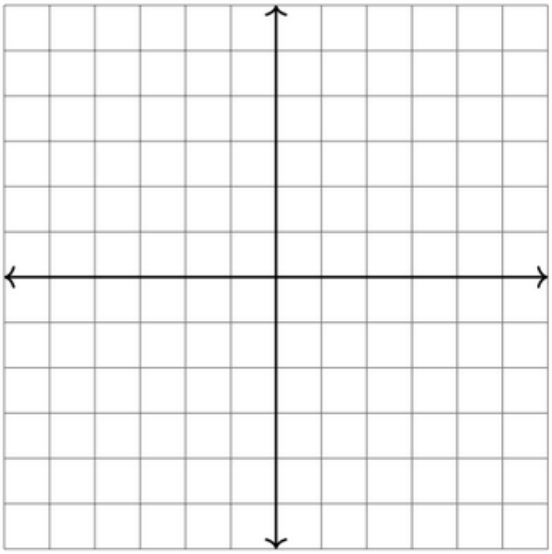
Write your answer in the box below:

$f^{-1}(2) =$

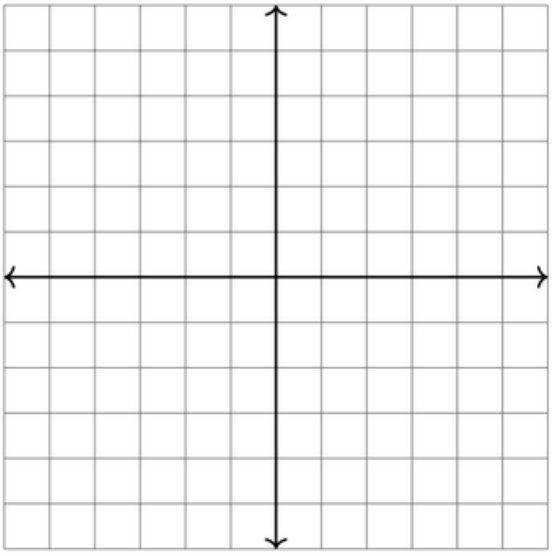
18. (4 points) Sketch the graph of $f(x) = -e^x + 2$. Label the horizontal asymptote and intercepts on your graph.



19. (4 points) Make a rough sketch of the graph $g(x) = x^3 - 2x^2 - 3x$. Label all intercepts on your graph.



20. (4 points) Graph the function $f(x) = -\log_3(x + 3)$ by transforming the graph of $y = \log_3(x)$. Label the vertical asymptote and all intercepts on your graph.



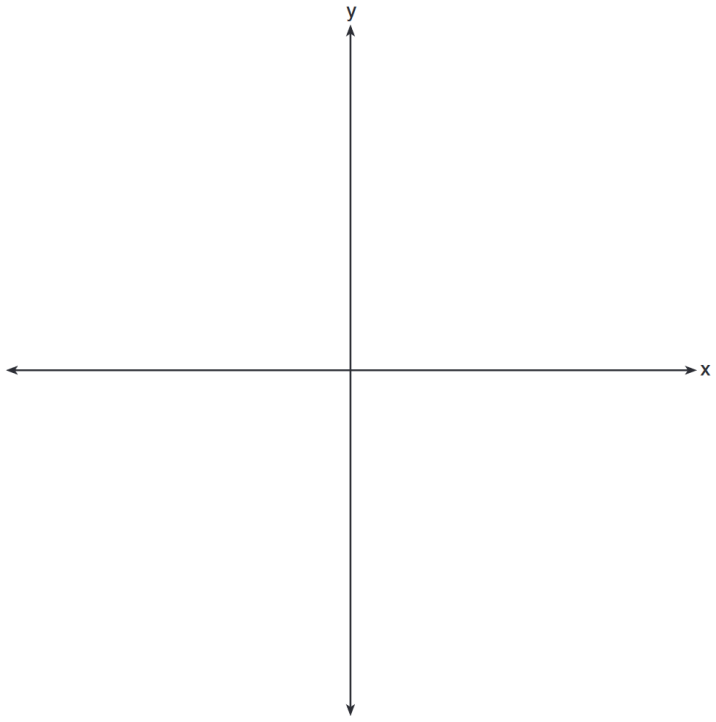
21. (4 points) Let $f(x) = -2\sin\left(x - \dfrac{\pi}{2}\right) + 1$.

(a) (½ point) Find the amplitude of $f(x)$.
Write your answer in the box below:

(b) (½ point) Find the period of $f(x)$.
Write your answer in the box below:

(c) (1 point) Find the horizontal shift b of $f(x)$.
Write your answer in the box below:

(d) (2 points) Sketch one complete period of the graph of $f(x)$ in the appropriate interval $\left[b, b + \dfrac{2\pi}{k}\right]$.



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