Division of Science | The City College of New York | Department of Mathematics

MATH195 – Precalculus

Summer 2024 Sample Final Exam

First Name:	Last Name:
EMPLID:	

Directions:

- **NO** notes, calculators, or other electronic devices allowed.

All electronic devices must be turned off and placed out of sight or they will be confiscated for the duration of the exam.

- Read each problem carefully. Unless otherwise instructed, be sure to show your work.
- Remember that it is your ***responsibility*** to answer each question clearly and in a way that convinces the grader that you understand how to solve each problem.

- GOOD LUCK!

Answer all 21 questions. ***You must show all of your work*** as neatly and clearly as possible and indicate the final answer in the box for each **non-graph** question. For all **graph** questions, you should sketch your graph on the grid provided.

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1. (8 points) Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$.

   (a) (4 points) Find the quotient and remainder when $P(x)$ is divided by $x + 2$.

Write your answer in the box below:

(b) (4 points) Use the Remainder Theorem to find $P(-2)$.

Write your answer in the box below:

$P(-2) =$

2. (8 points) The fox population in a certain region has a relative (continuous)growth rate of 8 percent per year. It is estimated that the population in the year 2000 was 400.

   (a) (4 points) Find a function that models the population t years after 2000 ($t = 0$ for 2000).

Write your answer in the box below:

   (b) (4 points) After how many years will the fox population reach 40,000?

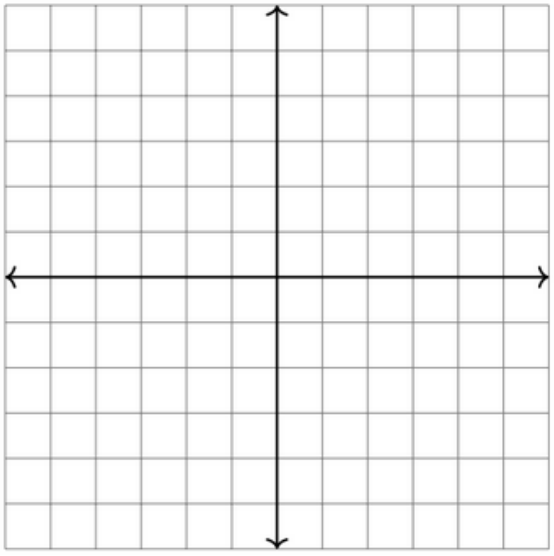
Write your answer in the box below:

3. (8 points) Consider the equation of the ellipse $25x^2 + 9y^2 - 100x - 54y - 44 = 0$.

(a) (4 points) Write the equation in standard form.

Write your answer in the box below:

(b) (4 points) Graph the ellipse on the axes below.



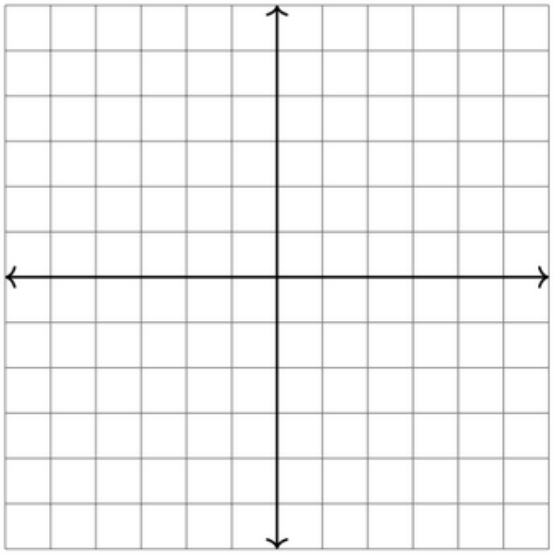
4. (8 points) Consider the system of nonlinear equations.

$$\begin{cases} x^2 + y^2 = 25 \\ x - y = 1 \end{cases}$$

(a) (4 points) Solve the system of equations. Write your answers in coordinate point form.

Write your answer in the box below:

(b) (4 points) Graph the system on the axes below. Clearly label the solutions to the system.



5. (4 points) Evaluate $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. Express your answer in radians.

Write your answer in the box below:

6. (4 points) Evaluate the difference quotient $\frac{f(2+h)-f(2)}{h}$ when $f(x) = x^2 + 1$.

Write your answer in the box below:

$\frac{f(2+h)-f(2)}{h} =$

7. (4 points) Find the average rate of change of $f(x) = \begin{cases} 1 & \text{if } x \leq -1 \\ 10 - x^2 & \text{if } x > 1 \end{cases}$ between $x = -5$ and $x = 2$

Write your answer in the box below:

8. (4 points) Find $\tan x$ if $\cos x = \frac{3}{5}$ and x is in Quadrant IV.

Write your answer in the box below:

$\tan x =$

9. (4 points) Solve the trigonometric equation $2\sin\theta + 1 = 0$, for all values of θ on the interval $0 \leq \theta \leq 2\pi$.

Write your answer in the box below:

$\theta =$

10. (4 points) Use the table below to find $f(g(5))$.

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	4	1	5	2	6

Write your answer in the box below:

$f(g(5)) =$

11. (4 points) Find the length s of a circular arc when $r = 12$ and $\theta = 30°$.

Write your answer in the box below:

$s =$

12. (4 points) Evaluate $\sin 15°$.

Write your answer in the box below:

$\sin 15° =$

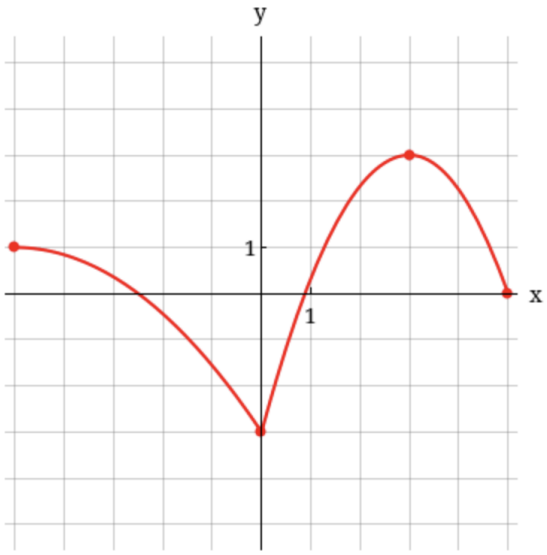
13. (4 points) Find an equation of the circle with center $(-1, 4)$ and passes through the point $(3, -2)$.

Write your answer in the box below:

14. (4 points) Solve the equation $2(10 + 2^x) = 100$ for x. Express your answer in terms of logarithms.

Write your answer in the box below:

15. (4 points) The graph of $y = f(x)$ is given below. Use the graph to find the range of f. Express your answer using interval notation.



Write your answer in the box below:

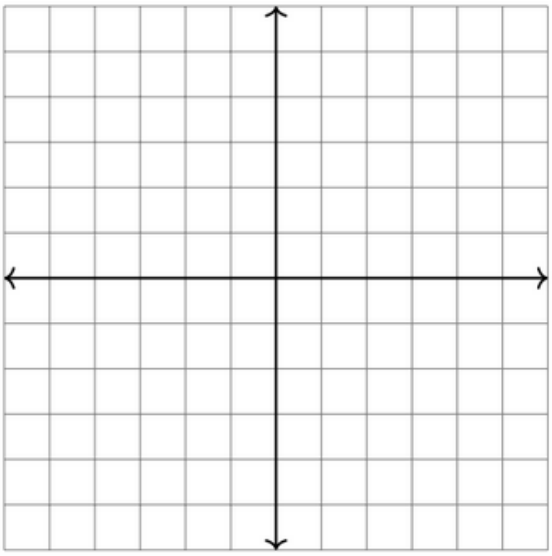
16. (4 points) Let $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 1$, find $f(g(x))$.

Write your answer in the box below:

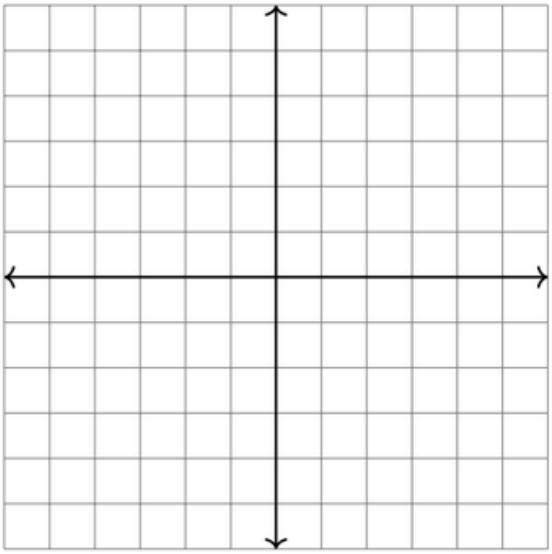
17. (4 points) Let $f(x) = \frac{x+1}{x}$. Find the inverse function $f^{-1}(x)$.

Write your answer in the box below:

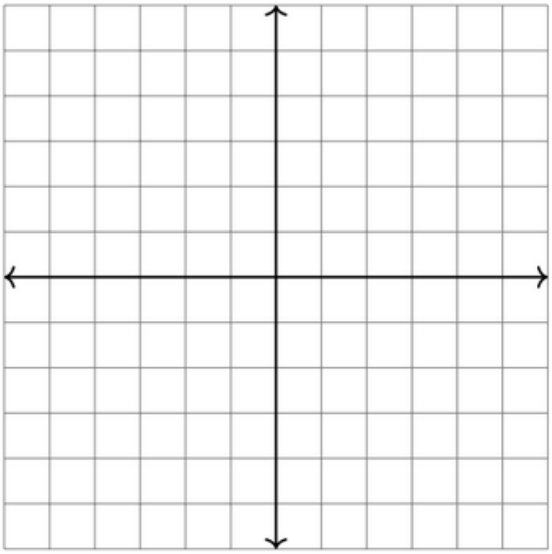
18. (4 points) Sketch the graph of $f(x) = -\log_2(x+1)$. Label the vertical asymptote and intercepts on your graph.



19. (4 points) Make a rough sketch of the graph $g(x) = -x^4 + x^2$. Label all intercepts on your graph.



20. (4 points) Graph the function $f(x) = -\sqrt{x+4}$ by transforming the graph of $y = \sqrt{x}$.



21. (4 points) Let $f(x) = 2\cos(x + \pi)$.

(a) (½ point) Find the amplitude of $f(x)$.
Write your answer in the box below:

(b) (½ point) Find the period of $f(x)$.
Write your answer in the box below:

(c) (1 point) Find the horizontal shift b of $f(x)$.
Write your answer in the box below:

(d) (2 points) Sketch one complete period of the graph of $f(x)$ in the appropriate interval $\left[b, b + \dfrac{2\pi}{k}\right]$.