PERCENTS - compliments of Dan Mosenkis

Percent Basics: Percents are connected to many ideas: fractions, decimals, proportions, relative amounts, and multiplicative change. You could say they are like the popular way of asking for ratings a few years ago: “On a scale of 1 to 10 …”, except they are on a scale of 0 to 100.

Percents emphasize relative amounts, rather than absolute. Suppose Joseph earned $20,000 and gave $2,000 to charity, while Bill earned $100,000 and gave $5,000 to charity. In absolute terms, Bill gave more; but in relative terms, Joseph gave more.

By definition, “per cent” means “÷100”, so 35% = 35/100 = .35 or 7/20 . Every percent is one way of writing a proportion. Other ways are fractions and decimals.

0	1/10	2/10		3/10	4/10	5/10	6/10	7/10		8/10	9/10	1	(fractions)
0		1/5	¼		2/5	½	3/5		¾	4/5		1	“
0	.1	.2	.25	.3	.4	.5	.6	.7	.75	.8	.9	1.0	(decimal)
0	10	20	25	30	40	50	60	70	75	80	90	100	(%)

Note that each percentage is 100 times the corresponding fraction and decimal; if you divide each percent by 100 and reduce it, you’ll get one of the fractions above it.
Or if you divide the percent by moving the decimal 2 places left, you’ll get the decimal version, and if you multiply the decimal by 100, you’d move the decimal right 2 places and get the percent.
Fraction to decimal: long division.
Decimal to percent: move point 2 places right (x100).
Percent to decimal: move point 2 places to left (÷100).

The simplest fact about percents is: (amount) = (proportion) x (base), and it permits answering simple questions. This equation can be solved for (proportion) = (amount)÷(base) , or for (base) = (amount) ÷ (proportion). I use “**proportion**” rather than “percent” to remind you that the proportion is a fraction or decimal; when written as __%, it appears to be 100 times bigger, because % means “÷ 100”.
For example: $30 is 60% of $50 ; so $30 = 60% x $50 = (60/100) x $50 = .60 x $50 .
Another: (a) Q: what is 45% of $300? A: amount = 45% x $300 = .45 x $300 = $135. (As a quick rough check, note that 45% is a bit less than half, so the answer should be a bit less than half of $300, which it is.)
Another: (b) Q: what percent of $300 is $45? A: (proportion) = (amount)÷(base) = $45/$300 = 15/100 = 15% (Rough check: $45 is far less than half of $300, and !5% is far less than 50%.)
Another: (c) Q: $300 is 45% of what number? A: (base) = (amount) ÷ (proportion) = $300/.45 = $666.67 .
NOTE: You don’t have to memorize three equations. In each case, you could start with the basic equation (amount) = (proportion) x (base), and solve from there:
(b) $45 = (pct) x $300 , so pct = $45/$300 = .15 = 15%
(c) $300 = .45 x (base), so (base) = $300/.45 =

Percents, deeper: When we compare two amounts, our first reaction is to see their difference, but percents are about seeing their ratio. For instance, if we compare $2,000 to $20,000 we might say that $2,000 is $18,000 less; the other way is to say it’s 1/10 the size, or 10%.

The equation (amount) = (proportion) x (base) shows that percents are about multiplication, but it goes deeper than that. If you buy something with a ticket price of $150 and must pay 8.5% tax, your first instinct is to figure the tax (multiply .085 by \$150), and then add it to \$150. But you could get the final amount in one step. How? Multiply \$150 by ____ (you figure it out!).

A good general method is to make a chart listing the absolute amounts opposite their percentages. Since percents are about proportions, one can find the missing number with a proportion.

In the following examples, one key to success is figuring out which of the three numbers (old salary, raise, new salary) should be the base, the 100%. Since percents are used for comparisons, the base of comparison is usually the first salary. A second key to success is figuring out the relations between all three dollar amounts (old salary + raise = new salary).

Example 1: John started out earning \$40,000. How much is he earning after a 14% raise?

0	\$40,000	←raise →	x = new salary	(amounts)
0	100	← 14 →	114	(percents)

Now set up an equation: x/40,000 = 114/100 (or, x/114 = 40,000/100).

Example 2: John is now earning \$40,000. How much did he earn before he got a 14% raise?

0	x = old salary	←raise →	\$40,000	(amounts)
0	100	← 14 →	114	(percents)

Now set up an equation: x/40,000 = 100/114 (or, x/100 = 40,000/114).

Note that there are many ways to set up a proportion. Some are right, some are wrong.

When you read the two numbers in the numerator or denominator, they should have something in common. Also, both numbers in each fraction should have something in common. In the first equation, x/40,000 = 100/114 , the x and 100 are in the same column, and so are the 40,000 and 114. Also, the x and the 40,000 are in the same row, and the same for the 100 and 114.

A similar analysis shows that the equation 100/x = 114/40,000 is correct.

But – if you wrote x/114 = 100/40,000 it would fail that test because the x and the 114 have nothing in common; they are in neither the same row nor the same column.

Note also that although several equations are correct, some are easier to solve than others:
To solve the equation $\frac{x}{100} = \frac{40{,}000}{114}$, if you cross-multiply, you would need one more step to solve for x. But you can solve it faster by just multiplying both sides by 100:
$x = \frac{40{,}000}{114} \times 100 = 35{,}087.72$. (Check: add 14% to 35,087.72 to get 40,000.)

Exercises: INTRODUCTORY PERCENTS

1. What is: (a) 40% of 800? (b) 400% of 800? (c) 0.4% of 800? (d) 1/4 % of 800?

2. (a) Convert 8% to a decimal; (b) convert 8% to a fraction.
(c) Convert .35 to a fraction; (d) convert .35 to a percentage.
(e) Convert 7/25 to a decimal; (f) convert 7/25 to a percentage.

3. (a) Convert ½ % to a decimal; (b) convert ½ % to a fraction.
(c) Convert 3.05 to a fraction; (d) convert 3.05 to a percentage.
(d) Convert 11/8 to a decimal; (f) convert 11/8 to a percentage.

4. Between 1980 and 1990, the price of the average tv set decreased from $200 to $120. What percentage decrease was that?

5. Between 1980 and 1990, the cost of a week's groceries for a family of 4 went from $120 to $200. What percent increase was that?

6. Suppose I paid $11,000 in taxes last year, and I had once remarked that I pay 30% of my income in taxes. What was my income?

7. Suppose Megasoft Corporation's earnings went from $200 million to $800 million. What percentage increase was that?

8. Suppose my income increased by 200% from 1990 to 1999, and I was making $30,000 in 1990; what would my 1999 income be?

9. Suppose my friend says his income increased by 300% since he started, and he's now making $50,000. What was his starting salary?

10. A bottle of laundry detergent holds 64 oz and sells for $9.60.
(a) What's the unit price (cost per ounce)?
(b) If the 64-oz size is increased by 20 oz., how much should the new size cost?
(c) If the 64-oz size is increased by 20%, how big will the new size be and how much should it cost?

11. A store routinely marks its items up 40% of what it paid. If it's selling a radio for $80, how much did the store pay for it?

12. A salesman offers you a radio for $100 and says that's a 20% discount. If he's telling the truth, what was the list price?

13. In December, a coat sold for $250. In January, the store raised all its prices 10%. But in February, you could take 10% off the sticker price in honor of our dead presidents.
(a) How much did it cost in January?
(b) How much did it cost in February?

14. I have a coupon good for 15% off at any Days Inn Motel. Their regular rate is $60, but there's a city entertainment and travel tax of 15%. How much will I pay?

PERCENT POINTERS AND HINTS:

Percent pointers:
If a quantity increases 24%, its new value is 124% of its old value.
If a quantity decreases 24%, its new value is 76% of its old value.
If a quantity increases 24%, you can find the new value by multiplying by 1.24 (representing 124%, the new value).
If a quantity decreases 24%, multiply by .76 to get the new value.
If a quantity is 324% of its old value, it has increased from 100% to 324%, so it has increased 224%.
If a quantity is 20% of its old value, it has decreased from 100% to 20%, so it has decreased 80%.

Percent Principles and Hints: (a) In percent problems, first ask: What's the BASE (the 100%) to which everything is compared? (20% of what?)
(b) Where time is involved, the base is usually the earlier number, but the base can change. When you're figuring the profit, the base is one thing; when you're figuring the discount, it's another. If a quantity increases, and then decreases by the same amount, the percent increase and decrease are not the same, since the base has changed. (Example: A store buys a coat for $60, prices it for $100, but ends up selling it for $80. In figuring the markup from 60 to 100, the base is 60; in considering the discount, from 100 to 80, the base is 100; in figuring the actual profit, from 60 to 80, the base is 60.)
(c) When several amounts are involved, be sure you understand the relations between them. (For example, what's the relation between wholesale price, selling price, and profit [or markup]?) Be sure you understand the relations between all these terms: buying price, selling price, asking price, profit, loss, discount. In particular, you should be able to complete these equations:
profit = ; loss = ; discount =
(d) Draw the picture (or the chart) and do the proportion.
(e) Percents are really about multiplying, not adding (even when it seems like we're adding --as in profit or markup-- or subtracting --as in loss or discount-- we're really multiplying).

Percent Problems II:

1. For every dollar a man makes, a woman makes 75 cents.
   A woman makes ____% less than a man.
   A man makes ____% more than a woman.

2. A storekeeper pays $400 (wholesale) for a TV. She marks its selling price as $600, but puts it on sale for $500.
   (a) If it had sold for $600, what percent profit would she have made?
   (b) What percent discount did the buyer (who paid $500 instead of $600) get?
   (c) What percent profit did the storekeeper end up getting?

3. A storekeeper has read that a 40% markup from wholesale is common, and tickets all her merchandise accordingly. But business is not too good; she decides that she could manage to do OK with a 30% markup. So she decides to run a one-month sale. What percent discount should she offer, so as to end up with a 30% profit?
   Illustrate your answer with an example of a fancy TV she paid $600 for. What was the original selling price she put on the ticket? Show how the %-off sale reduced the price to a new, sale price. What percent discount did the buyer get? Did the storekeeper really end up making her 30% profit?

4. A retiree needs $10,000 in cash for some upcoming expenses. He intends to withdraw it from his IRA, but he knows that the bank is required to withhold 20% of any withdrawal for Federal taxes. How much should he request as a withdrawal?

5. A builder of a housing development wants to order several thousand boards to be used as exterior siding. He wants the boards to be 10" wide. The lumberyard tells him that they cut the boards while still moist, and he should allow for shrinkage of 20% in width as they dry. So he adds 20% to his width, and orders them 12" wide. When he gets them, he finds that almost all are less than 10" wide, and he sues the lumberyard. He loses. Why? Where did he go wrong? What should he have done?

6. The regular price for a TV is $450, but it's on sale for $360.
   a. The regular price is ___ percent more than the sale price.
   b. The regular price is ___ percent of the sale price.
   c. The sale price is _____ percent less than the regular price.
   d. The sale price is _____ percent of the sale price.
7. When our school required its students to wear uniforms, the pass rate in math increased from 50 percent to 60 percent.
   a. The new pass rate is _____ percentage points above the old rate.
   b. The new rate is _____ percent more than the old rate.
   c. The new pass rate is _____ percent of the old pass rate.

8. Make up numbers to illustrate the following:
   a. Jeff's income increased by 50 percent from last year to this.
   b. Tina's income increased by 200 percent from last year to this.
   c. Mike's income decreased by 50 percent from last year to this.
   d. Sara's income decreased by 200 percent from last year to this.

9. Find the amount of profit, if assets were $10,000:
   a. Profits were .25 of assets.
   b. Profits were .25% of assets.
   c. Profits were 25% of assets.

10. Jack weighs 40% more than Jill, so Jill weighs __% less than Jack.

11. Assume the cutoff for DWI is .1%. If Sam's blood alcohol concentration is .13%, his BAC is:
   a. ___ percentage points above the limit.
   b. ___ percent above the legal limit.

12. For every 400 Hondas sold, there were 500 Toyotas sold.
   a. Honda sold ___% fewer than Toyota.
   b. Toyota sold ___% more than Honda.