Note: Review examples1 to 9 in the text for guidance.

Note: When adding and subtracting radicals, treat each different radicals as if they are different variables. For example: The statement $4\sqrt{3}+2\sqrt{5}$ is already in simplified forma and we cannot complete the operation because let $x=\sqrt{3}$ and $y=\sqrt{5}$. Then our original statement is same as $4\sqrt{3}+2\sqrt{5} \quad \Rightarrow \quad 4x+2y$.

Additional examples

2) $\sqrt{6}\sqrt{2}$

$\sqrt{6}\sqrt{2}=(\sqrt{3}\sqrt{2})\sqrt{2}=2\sqrt{3}$

4) $(3\sqrt{5})(2\sqrt{7})$

$(3\sqrt{5})(2\sqrt{7})=(3)(2)\sqrt{5}\sqrt{7}=6\sqrt{35}$

6) $(4\sqrt{35})(2\sqrt{21})(5\sqrt{15})$

$$(4\sqrt{35})(2\sqrt{21})(5\sqrt{15})=(4\sqrt{5}\sqrt{7})(2\sqrt{3}\sqrt{7})(5\sqrt{3}\sqrt{5})=(4)(2)(5)(\sqrt{3}\sqrt{3})(\sqrt{5}\sqrt{5})(\sqrt{7}\sqrt{7})$$
$$=(2)(5)(4)(3)(5)(7)=(10)(4)(3)(35)=(10)(4)(105)=(10)(420)=4200$$

8) $(2\sqrt[3]{2})(6\sqrt[3]{4})$

$$(2\sqrt[3]{2})(6\sqrt[3]{4})=(2\sqrt[3]{(2)^1})(6\sqrt[3]{(2)^2})=(2)(6)(\sqrt[3]{(2)^1}\sqrt[3]{(2)^2})=12\sqrt[3]{(2)^3}=12(2)=24$$

10) $\sqrt{2}(5\sqrt{3}+4\sqrt{2})$

$\sqrt{2}(5\sqrt{3}+4\sqrt{2})=5\sqrt{3}\sqrt{2}+4\sqrt{2}\sqrt{2}=5\sqrt{6}+4(2)=5\sqrt{6}+8$

12) $7\sqrt[3]{5}(3\sqrt[3]{25}-2)$

$$7\sqrt[3]{5}(3\sqrt[3]{25}-2)=7\sqrt[3]{(5)^1}(3\sqrt[3]{(5)^2}-2)=(7)(3)\sqrt[3]{(5)^1}\sqrt[3]{(5)^2}-(7)(2)\sqrt[3]{(5)^1}=21(5)-14\sqrt[3]{5}=105-14\sqrt[3]{5}$$

14) $(\sqrt{5}-\sqrt{2})(3\sqrt{5}+2\sqrt{2})$

$$(\sqrt{5}-\sqrt{2})(3\sqrt{5}+2\sqrt{2})=3\sqrt{5}\sqrt{5}+2\sqrt{5}\sqrt{2}-3\sqrt{5}\sqrt{2}-2\sqrt{2}\sqrt{2}=3(5)+2\sqrt{10}-3\sqrt{10}-2(2)$$
$$=15-\sqrt{10}-4=11-\sqrt{10}$$

16) $(\sqrt{x}+4)(\sqrt{x}+2)$

$(\sqrt{x}+4)(\sqrt{x}+2)=\sqrt{x}\sqrt{x}+2\sqrt{x}+4\sqrt{x}+(4)(2)=x+6\sqrt{x}+8$

18) $(\sqrt{7}-3\sqrt{3})(2\sqrt{7}-4\sqrt{3})$

$$(\sqrt{7}-3\sqrt{3})(2\sqrt{7}-4\sqrt{3})=2\sqrt{7}\sqrt{7}-4\sqrt{7}\sqrt{3}-(3)(2)\sqrt{7}\sqrt{3}+(3)(4)\sqrt{3}\sqrt{3}=2(7)-4\sqrt{21}-6\sqrt{21}+(12)(3)$$
$$=14-10\sqrt{21}+36=50-10\sqrt{21}$$

20) $(\sqrt{5}-2)^2$

$(\sqrt{5}-2)^2 = (\sqrt{5}-2)(\sqrt{5}-2) = \sqrt{5}\sqrt{5} - 2\sqrt{5} - 2\sqrt{5} + (2)(2) = 5 - 4\sqrt{5} + 4 = 9 - 4\sqrt{5}$

22) $(\sqrt{x}+4)^2$

$(\sqrt{x}+4)^2 = (\sqrt{x}+4)(\sqrt{x}+4) = \sqrt{x}\sqrt{x} + 4\sqrt{x} + 4\sqrt{x} + (4)(4) = x + 8\sqrt{x} + 16$

24) $(5\sqrt{a}-2\sqrt{b})^2$

$(5\sqrt{a}-2\sqrt{b})^2 = (5\sqrt{a}-2\sqrt{b})(5\sqrt{a}-2\sqrt{b}) = (5)(5)\sqrt{a}\sqrt{a} - (5)(2)\sqrt{a}\sqrt{b} - (5)(2)\sqrt{a}\sqrt{b} + (2)(2)\sqrt{b}\sqrt{b}$

$= 25a - 10\sqrt{ab} - 10\sqrt{ab} + 4b = 25a - 20\sqrt{ab} + 4b$

26) $(\sqrt{x-3}+2)^2$

$(\sqrt{x-3}+2)^2 = (\sqrt{x-3}+2)(\sqrt{x-3}+2) = \sqrt{x-3}\sqrt{x-3} + 2\sqrt{x-3} + 2\sqrt{x-3} + (2)(2)$

$= (x-3) + 4\sqrt{x-3} + 4 = x + 1 + 4\sqrt{x-3}$

28) $(\sqrt{x-3}-4)^2$

$(\sqrt{x-3}-4)^2 = (\sqrt{x-3}-4)(\sqrt{x-3}-4) = \sqrt{x-3}\sqrt{x-3} - 4\sqrt{x-3} - 4\sqrt{x-3} + (4)(4)$

$= (x-3) - 8\sqrt{x-3} + 16 = x + 13 - 8\sqrt{x-3}$

30) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = \sqrt{5}\sqrt{5} + \sqrt{5}\sqrt{2} - \sqrt{5}\sqrt{2} - \sqrt{2}\sqrt{2} = 5 + \sqrt{10} - \sqrt{10} - 2 = 3$

32) $(\sqrt{a}+5)(\sqrt{a}-5)$

$(\sqrt{a}+5)(\sqrt{a}-5) = \sqrt{a}\sqrt{a} - 5\sqrt{a} + 5\sqrt{a} - (5)(5) = a - 25$

34) $(3-\sqrt{x})(3+\sqrt{x})$

$(3-\sqrt{x})(3+\sqrt{x}) = (3)(3) + 3\sqrt{x} - 3\sqrt{x} - \sqrt{x}\sqrt{x} = 9 - x$

36) $(\sqrt{x+3}+5)(\sqrt{x+3}-5)$

$(\sqrt{x+3}+5)(\sqrt{x+3}-5) = \sqrt{x+3}\sqrt{x+3} - 5\sqrt{x+3} + 5\sqrt{x+3} - (5)(5) = (x+3) - 25 = x - 23$

38) $(\sqrt{5}-2)^3$

$(\sqrt{5}-2)^3 = (\sqrt{5}-2)\{(\sqrt{5}-2)(\sqrt{5}-2)\} = (\sqrt{5}-2)\{\sqrt{5}\sqrt{5} - 2\sqrt{5} - 2\sqrt{5} + (2)(2)\} = (\sqrt{5}-2)\{5 - 4\sqrt{5} + 4\}$

$= (\sqrt{5}-2)\{9 - 4\sqrt{5}\} = 9\sqrt{5} - 4\sqrt{5}\sqrt{5} - (2)(9) + (2)(4)\sqrt{5} = 9\sqrt{5} - 4(5) - 18 + 8\sqrt{5}$

$= 17\sqrt{5} - 20 - 18 = 17\sqrt{5} - 38$

40) $\dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

$$\frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}=\left(\frac{\sqrt{5}}{(\sqrt{5}+\sqrt{3})}\right)\left(\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}\right)=\frac{\sqrt{5}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\frac{\sqrt{5}\sqrt{5}-\sqrt{5}\sqrt{3}}{\sqrt{5}\sqrt{5}-\sqrt{5}\sqrt{3}+\sqrt{5}\sqrt{3}-\sqrt{3}\sqrt{3}}$$

$$=\frac{5-\sqrt{15}}{5-\sqrt{15}+\sqrt{15}-3}=\frac{5-\sqrt{15}}{5-3}=\frac{5-\sqrt{15}}{2}$$

42) $\dfrac{\sqrt{7}}{\sqrt{7}+1}$

$$\frac{\sqrt{7}}{\sqrt{7}+1}=\left(\frac{\sqrt{7}}{(\sqrt{7}+1)}\right)\left(\frac{(\sqrt{7}-1)}{(\sqrt{7}-1)}\right)=\frac{\sqrt{7}(\sqrt{7}-1)}{(\sqrt{7}+1)(\sqrt{7}-1)}=\frac{\sqrt{7}\sqrt{7}-\sqrt{7}}{\sqrt{7}\sqrt{7}-\sqrt{7}+\sqrt{7}-1}=\frac{7-\sqrt{7}}{7-1}=\frac{7-\sqrt{7}}{6}$$

44) $\dfrac{\sqrt{x}}{\sqrt{x}+2}$

$$\frac{\sqrt{x}}{\sqrt{x}+2}=\left(\frac{\sqrt{x}}{(\sqrt{x}+2)}\right)\left(\frac{(\sqrt{x}-2)}{(\sqrt{x}-2)}\right)=\frac{\sqrt{x}(\sqrt{x}-2)}{(\sqrt{x}+2)(\sqrt{x}-2)}=\frac{\sqrt{x}\sqrt{x}-2\sqrt{x}}{\sqrt{x}\sqrt{x}-2\sqrt{x}+2\sqrt{x}-(2)(2)}=\frac{x-2\sqrt{x}}{x-4}$$

46) $\dfrac{\sqrt{7}}{3\sqrt{7}-2}$

$$\frac{\sqrt{7}}{3\sqrt{7}-2}=\left(\frac{\sqrt{7}}{(3\sqrt{7}-2)}\right)\left(\frac{(3\sqrt{7}+2)}{(3\sqrt{7}+2)}\right)=\frac{\sqrt{7}(3\sqrt{7}+2)}{(3\sqrt{7}-2)(3\sqrt{7}+2)}=\frac{3\sqrt{7}\sqrt{7}+2\sqrt{7}}{(3)(3)\sqrt{7}\sqrt{7}+(3)(2)\sqrt{7}-(3)(2)\sqrt{7}-(2)(2)}$$

$$=\frac{3(7)+2\sqrt{7}}{(9)(7)+6\sqrt{7}-6\sqrt{7}-4}=\frac{21+2\sqrt{7}}{63-4}=\frac{21+2\sqrt{7}}{59}$$

48) $\dfrac{2}{\sqrt{x}+\sqrt{y}}$

$$\frac{2}{\sqrt{x}+\sqrt{y}}=\left(\frac{2}{(\sqrt{x}+\sqrt{y})}\right)\left(\frac{(\sqrt{x}-\sqrt{y})}{(\sqrt{x}-\sqrt{y})}\right)=\frac{2(\sqrt{x}-\sqrt{y})}{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}=\frac{2\sqrt{x}-2\sqrt{y}}{\sqrt{x}\sqrt{x}-\sqrt{x}\sqrt{y}+\sqrt{x}\sqrt{y}-\sqrt{y}\sqrt{y}}$$

$$=\frac{2\sqrt{x}-2\sqrt{y}}{x-\sqrt{xy}+\sqrt{xy}-y}=\frac{2\sqrt{x}-2\sqrt{y}}{x-y}$$

50) $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\left(\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}\right)\left(\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}\right)=\frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\frac{\sqrt{5}\sqrt{5}-\sqrt{5}\sqrt{3}-\sqrt{5}\sqrt{3}+\sqrt{3}\sqrt{3}}{\sqrt{5}\sqrt{5}-\sqrt{5}\sqrt{3}+\sqrt{5}\sqrt{3}-\sqrt{3}\sqrt{3}}$$

$$=\frac{5-\sqrt{15}-\sqrt{15}+3}{5-\sqrt{15}+\sqrt{15}-3}=\frac{8-2\sqrt{15}}{5-3}=\frac{8-2\sqrt{15}}{2}=\frac{2(4-\sqrt{15})}{2}=\frac{(4-\sqrt{15})}{1}=4-\sqrt{15}$$

52) $\dfrac{\sqrt{11}+3}{\sqrt{11}-3}$

$$\frac{\sqrt{11}+3}{\sqrt{11}-3}=\left(\frac{(\sqrt{11}+3)}{(\sqrt{11}-3)}\right)\left(\frac{(\sqrt{11}+3)}{(\sqrt{11}+3)}\right)=\frac{(\sqrt{11}+3)(\sqrt{11}+3)}{(\sqrt{11}-3)(\sqrt{11}+3)}=\frac{\sqrt{11}\sqrt{11}+3\sqrt{11}+3\sqrt{11}+(3)(3)}{\sqrt{11}\sqrt{11}+3\sqrt{11}-3\sqrt{11}-(3)(3)}$$

$$=\frac{11+3\sqrt{11}+3\sqrt{11}+9}{11-9}=\frac{20+6\sqrt{11}}{2}=\frac{2(10+3\sqrt{11})}{2}=\frac{(10+3\sqrt{11})}{1}=10+3\sqrt{11}$$

54) $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

$$\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\left(\frac{(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})}\right)\left(\frac{(\sqrt{a}-\sqrt{b})}{(\sqrt{a}-\sqrt{b})}\right)=\frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}=\frac{\sqrt{a}\sqrt{a}-\sqrt{a}\sqrt{b}-\sqrt{a}\sqrt{b}+\sqrt{b}\sqrt{b}}{\sqrt{a}\sqrt{a}-\sqrt{a}\sqrt{b}+\sqrt{a}\sqrt{b}-\sqrt{b}\sqrt{b}}$$

$$=\frac{a-\sqrt{ab}-\sqrt{ab}+b}{a-\sqrt{ab}+\sqrt{ab}-b}=\frac{a-2\sqrt{ab}+b}{a-b}$$

56) $\dfrac{\sqrt{x}-3}{\sqrt{x}+3}$

$$\frac{\sqrt{x}-3}{\sqrt{x}+3}=\left(\frac{(\sqrt{x}-3)}{(\sqrt{x}+3)}\right)\left(\frac{(\sqrt{x}-3)}{(\sqrt{x}-3)}\right)=\frac{(\sqrt{x}-3)(\sqrt{x}-3)}{(\sqrt{x}+3)(\sqrt{x}-3)}=\frac{\sqrt{x}\sqrt{x}-3\sqrt{x}-3\sqrt{x}+(3)(3)}{\sqrt{x}\sqrt{x}-3\sqrt{x}+3\sqrt{x}-(3)(3)}=\frac{x-6\sqrt{x}+9}{x-9}$$

58) $\dfrac{5\sqrt{6}+2\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$$\frac{5\sqrt{6}+2\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\left(\frac{(5\sqrt{6}+2\sqrt{2})}{(\sqrt{6}-\sqrt{2})}\right)\left(\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}\right)=\frac{(5\sqrt{6}+2\sqrt{2})(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$$

$$=\frac{5\sqrt{6}\sqrt{6}+5\sqrt{6}\sqrt{2}+2\sqrt{6}\sqrt{2}+2\sqrt{2}\sqrt{2}}{\sqrt{6}\sqrt{6}+\sqrt{6}\sqrt{2}-\sqrt{6}\sqrt{2}-\sqrt{2}\sqrt{2}}=\frac{5(6)+5\sqrt{12}+2\sqrt{12}+2(4)}{6+\sqrt{12}-\sqrt{12}-2}$$

$$=\frac{30+7\sqrt{12}+8}{6-2}=\frac{38+7\sqrt{4}\sqrt{3}}{4}=\frac{38+7(2)\sqrt{3}}{4}=\frac{2(17+7\sqrt{3})}{4}=\frac{17+7\sqrt{3}}{2}$$

60) $\dfrac{5\sqrt{x}-1}{2+\sqrt{x}}$

Method 1:

$$\frac{5\sqrt{x}-1}{2+\sqrt{x}}=\left(\frac{(5\sqrt{x}-1)}{(2+\sqrt{x})}\right)\left(\frac{(2-\sqrt{x})}{(2-\sqrt{x})}\right)=\frac{(5\sqrt{x}-1)(2-\sqrt{x})}{(2+\sqrt{x})(2-\sqrt{x})}$$

$$=\frac{(5)(2)\sqrt{x}-5\sqrt{x}\sqrt{x}-(1)(2)+\sqrt{x}}{(2)(2)-2\sqrt{x}+2\sqrt{x}-\sqrt{x}\sqrt{x}}=\frac{10\sqrt{x}-5x-2+\sqrt{x}}{4-x}=\frac{11\sqrt{x}-5x-2}{4-x}$$

Method 2:

$$\frac{5\sqrt{x}-1}{2+\sqrt{x}}=\frac{5\sqrt{x}-1}{\sqrt{x}+2}=\left(\frac{(5\sqrt{x}-1)}{(\sqrt{x}+2)}\right)\left(\frac{(\sqrt{x}-2)}{(\sqrt{x}-2)}\right)=\frac{(5\sqrt{x}-1)(\sqrt{x}-2)}{(\sqrt{x}+2)(\sqrt{x}-2)}$$

$$=\frac{5\sqrt{x}\sqrt{x}-(5)(2)\sqrt{x}-\sqrt{x}+2}{\sqrt{x}\sqrt{x}-2\sqrt{x}+2\sqrt{x}-(2)(2)}=\frac{5x-10\sqrt{x}-\sqrt{x}+2}{x-4}=\frac{5x-11\sqrt{x}+2}{x-4}$$