Property 1 For Radicals: $\sqrt[n]{ab} = (\sqrt[n]{a})(\sqrt[n]{b})$

Property 2 For Radicals: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad b \neq 0$

Simplified Form for Radical Expressions:

1. None of the factors on the radicand (the quantity under the radical sign) can be written as powers greater than or equal to the index—that is, no perfect squares can be factors of the quantity under a square root sign, no perfect cubes can be factors of that is under a cube root sign, and so forth.
2. There are no fractions under the radical sign.
3. There are no radicals in the denominator (this is true for a beginning elementary algebra course. There will be many instances where it will be an advantage to keep radical expression in the denominator).

Property 3 For Radicals:

If a is a real number, then $\sqrt{a^2} = |a|$

Extending Property 3 For Radicals:

If a is a real number, then $\begin{cases} \sqrt[n]{a^n} = |a| & \text{if} \quad n \text{ is even} \\ \sqrt[n]{a^n} = a & \text{if} \quad n \text{ is odd} \end{cases}$

Property 3-a For Radicals:

If a is a positive real number, then $\left(\sqrt{a}\right)^2 = a$

Extending Property 3-a For Radicals:

If a is a positive real number and n is even, then $\left(\sqrt[n]{a}\right)^n = a$

If a is a real number and n is odd, then $\left(\sqrt[n]{a}\right)^n = a$

Note: Review all examples in the text for guidance.

Additional examples

2) $\sqrt{32}$

$\sqrt{32} = \sqrt{(16)(2)} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

4) $\sqrt{75}$

$\sqrt{75} = \sqrt{(25)(3)} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

6) $\sqrt{128}$

$\sqrt{128} = \sqrt{(64)(2)} = \sqrt{64}\sqrt{2} = 8\sqrt{2}$

8) $\sqrt{200}$

$\sqrt{200} = \sqrt{(100)(2)} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$

10) $\sqrt{27}$

$\sqrt{27} = \sqrt{(9)(3)} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

12) $\sqrt{972}$

	972
2	486
2	243
3	81
3	27
3	9
3	3
3	1

$\Rightarrow$ $972 = (2)(2)(3)(3)(3)(3)(3)$
$= (2)^2(3)^5$

$\sqrt{972} = \sqrt{(2)^2(3)^5} = \sqrt{(2)^2(3)^4(3)}$
$= \sqrt{(2)^2(3)^4}\sqrt{3} = (2)(3)^2\sqrt{3}$
$= 18\sqrt{3}$

14) $\sqrt[3]{24}$

$\sqrt[3]{24} = \sqrt[3]{(8)(3)} = \sqrt[3]{8}\sqrt[3]{3} = 2\sqrt[3]{3}$

16) $\sqrt[3]{162}$

$\sqrt[3]{162} = \sqrt[3]{(81)(2)} = \sqrt[3]{(27)(3)(2)} = \sqrt[3]{27}\sqrt[3]{(3)(2)} = 3\sqrt[3]{6}$

18) $\sqrt[3]{1536}$

	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
2	3
3	1

$\Rightarrow$ $1536 = (2)(2)(2)(2)(2)(2)(2)(2)(2)(3)$
$= (2)^9(3)$

$\sqrt[3]{1536} = \sqrt[3]{(2)^9(3)}$
$= \sqrt[3]{(2)^9}\sqrt[3]{(3)}$
$= \left((2)^3\right)\sqrt[3]{3}$
$= 8\sqrt[3]{3}$

20) $\sqrt[4]{48}$

$\sqrt[4]{48} = \sqrt[4]{(16)(3)} = \sqrt[4]{16}\sqrt[4]{3} = 2\sqrt[4]{3}$

22) $\sqrt{27x^5}$

$$\sqrt{27x^5} = \sqrt{(9x^4)(3x)} = \sqrt{9x^4}\sqrt{3x} = 3x^2\sqrt{3x}$$

24) $\sqrt[5]{32y^7}$

$$\sqrt[5]{32y^7} = \sqrt[5]{(32y^5)(y^2)} = \sqrt[5]{32y^5}\sqrt[5]{y^2} = 2y\sqrt[5]{y^2}$$

26) $\sqrt[3]{128x^6y^2}$

$$\begin{array}{r|l} & 128 \\ \hline 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ 2 & 1 \end{array} \Rightarrow \begin{aligned} 972 &= (2)(2)(2)(2)(2)(2)(2) \\ &= (2)^7 \end{aligned}$$

$$\begin{aligned} \sqrt[3]{128x^6y^2} &= \sqrt[3]{(2)^7x^6y^2} \\ &= \sqrt[3]{\left((2)^6x^6\right)\left(2y^2\right)} \\ &= \sqrt[3]{(2)^6x^6}\sqrt[3]{\left(2y^2\right)} \\ &= \left((2)^2x^2\right)\sqrt[3]{2y^2} \\ &= 4x^2\sqrt[3]{2y^2} \end{aligned}$$

28) $\sqrt{72a^4b^3c^2}$

$$\sqrt{72a^4b^3c^2} = \sqrt{(36a^4b^2c^2)(2b)} = \sqrt{36a^4b^2c^2}\sqrt{2b} = 6a^2bc\sqrt{2b}$$

30) $\sqrt[3]{72a^4b^3c^2}$

$$\sqrt[3]{72a^4b^3c^2} = \sqrt[3]{(8a^3b^3)(9ac^2)} = \sqrt[3]{8a^3b^3}\sqrt[3]{9ac^2} = 2ab\sqrt[3]{9ac^2}$$

32) $\sqrt[4]{32x^9y^{10}}$

$$\sqrt[4]{32x^9y^{10}} = \sqrt[4]{(16x^8y^8)(2xy^2)} = \sqrt[4]{16x^8y^8}\sqrt[4]{2xy^2} = 2x^2y^2\sqrt[4]{2xy^2}$$

34) $\sqrt[5]{64x^8y^4z^{11}}$

$$\sqrt[5]{64x^8y^4z^{11}} = \sqrt[5]{(32x^5z^{10})(2x^3y^4z)} = \sqrt[5]{32x^5z^{10}}\sqrt[4]{2x^3y^4z} = 2xz^2\sqrt[5]{2x^3y^4z}$$

42) $\dfrac{3}{\sqrt{2}}$

$$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{3\sqrt{2}}{2}$$

44) $\dfrac{7}{\sqrt{5}}$

$$\frac{7}{\sqrt{5}}=\frac{7}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{7\sqrt{5}}{5}$$

46) $\sqrt{\dfrac{1}{3}}$

$$\sqrt{\frac{1}{3}}=\frac{\sqrt{1}}{\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{3}}{3}$$

48) $\sqrt{\dfrac{1}{6}}$

$$\sqrt{\frac{1}{6}}=\frac{\sqrt{1}}{\sqrt{6}}=\frac{1}{\sqrt{6}}=\frac{1}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)=\frac{\sqrt{6}}{6}$$

50) $\dfrac{5}{\sqrt[3]{3}}$

$$\frac{5}{\sqrt[3]{3}}=\frac{5}{\sqrt[3]{(3)^1}}\left(\frac{\left(\sqrt[3]{(3)^2}\right)}{\left(\sqrt[3]{(3)^2}\right)}\right)=\frac{5\sqrt[3]{(3)^2}}{3}=\frac{5\sqrt[3]{9}}{3}$$

52) $\dfrac{3}{\sqrt[3]{4}}$

$$\frac{3}{\sqrt[3]{4}}=\frac{3}{\sqrt[3]{(2)^2}}\left(\frac{\left(\sqrt[3]{(2)^1}\right)}{\left(\sqrt[3]{(2)^1}\right)}\right)=\frac{3\sqrt[3]{(2)^1}}{2}=\frac{3\sqrt[3]{2}}{2}$$

54) $\sqrt[4]{\dfrac{5}{3x^2}}$

$$\sqrt[4]{\frac{5}{3x^2}}=\frac{\sqrt[4]{5}}{\sqrt[4]{3x^2}}=\frac{\sqrt[4]{5}}{\sqrt[4]{(3)^1x^2}}\left(\frac{\left(\sqrt[4]{(3)^3x^2}\right)}{\left(\sqrt[4]{(3)^3x^2}\right)}\right)=\frac{\sqrt[4]{5}\left(\sqrt[4]{(3)^3x^2}\right)}{3x}=\frac{\sqrt[4]{5(27)x^2}}{3x}=\frac{\sqrt[4]{135x^2}}{3x}$$

56) $\sqrt[4]{\dfrac{27}{y}}$

$$\sqrt[4]{\frac{27}{y}}=\frac{\sqrt[4]{27}}{\sqrt[4]{y}}=\frac{\sqrt[4]{27}}{\sqrt[4]{y^1}}\left(\frac{\left(\sqrt[4]{y^3}\right)}{\left(\sqrt[4]{y^3}\right)}\right)=\frac{\sqrt[4]{27}\left(\sqrt[4]{y^3}\right)}{y}=\frac{\sqrt[4]{27y^3}}{y}$$

58) $\sqrt[3]{\dfrac{7x}{6y}}$

$$\sqrt[3]{\frac{7x}{6y}}=\frac{\sqrt[3]{7x}}{\sqrt[3]{6y}}=\frac{\sqrt[3]{7x}}{\sqrt[3]{(6)^1y^1}}\left(\frac{\left(\sqrt[3]{(6)^2y^2}\right)}{\left(\sqrt[3]{(6)^2y^2}\right)}\right)=\frac{\sqrt[3]{7x}\left(\sqrt[3]{(6)^2y^2}\right)}{6y}=\frac{\sqrt[3]{(7x)(36y^2)}}{6y}=\frac{\sqrt[3]{252xy^2}}{6y}$$

60) $\sqrt[3]{\dfrac{5x}{4y}}$

$$\sqrt[3]{\frac{5x}{4y}}=\frac{\sqrt[3]{5x}}{\sqrt[3]{4y}}=\frac{\sqrt[3]{5x}}{\sqrt[3]{(2)^1y^1}}\left(\frac{\left(\sqrt[3]{(2)^1y^2}\right)}{\left(\sqrt[3]{(2)^1y^2}\right)}\right)=\frac{\sqrt[3]{5x}\left(\sqrt[3]{(2)^1y^2}\right)}{2y}=\frac{\sqrt[3]{(5x)(2y^2)}}{2y}=\frac{\sqrt[3]{10xy^2}}{2y}$$

62) $\sqrt[4]{\dfrac{8}{9x^3}}$

$$\sqrt[4]{\frac{8}{9x^3}}=\frac{\sqrt[4]{8}}{\sqrt[4]{9x^3}}=\frac{\sqrt[4]{8}}{\sqrt[4]{(3)^2x^3}}\left(\frac{\left(\sqrt[4]{(3)^2x^1}\right)}{\left(\sqrt[4]{(3)^2x^1}\right)}\right)=\frac{\sqrt[4]{8}\left(\sqrt[4]{(3)^2x^1}\right)}{3x}=\frac{\sqrt[4]{(8)(9x)}}{3x}=\frac{\sqrt[4]{72x}}{3x}$$

64) $\sqrt{\dfrac{12x^5}{7y}}$

$$\sqrt{\frac{12x^5}{7y}}=\frac{\sqrt{12x^5}}{\sqrt{7y}}=\frac{\sqrt{(4x^4)(3x)}}{\sqrt{7y}}=\frac{\sqrt{4x^4}\sqrt{3x}}{\sqrt{7y}}=\frac{2x^2\sqrt{3x}}{\sqrt{7y}}=\frac{2x^2\sqrt{3x}}{\sqrt{7y}}\left(\frac{\sqrt{7y}}{\sqrt{7y}}\right)=\frac{2x^2\sqrt{3x}\sqrt{7y}}{7y}$$

$$=\frac{2x^2\sqrt{(3x)(7y)}}{7y}=\frac{2x^2\sqrt{21xy}}{7y}$$

66) $\sqrt{\dfrac{50x^2y^3}{3z}}$

$$\sqrt{\frac{50x^2y^3}{3z}}=\frac{\sqrt{50x^2y^3}}{\sqrt{3z}}=\frac{\sqrt{(25x^2y^2)(2y)}}{\sqrt{3z}}=\frac{\sqrt{25x^2y^2}\sqrt{2y}}{\sqrt{3z}}=\frac{5xy\sqrt{2y}}{\sqrt{3z}}=\frac{5xy\sqrt{2y}}{\sqrt{3z}}\left(\frac{\sqrt{3z}}{\sqrt{3z}}\right)=\frac{5xy\sqrt{2y}\sqrt{3z}}{3z}$$

$$=\frac{5xy\sqrt{(2y)(3z)}}{3z}=\frac{5xy\sqrt{6yz}}{3z}$$

68) $\sqrt[3]{\dfrac{54a^5b^4}{25c^2}}$

$$\sqrt[3]{\frac{54a^5b^4}{25c^2}} = \frac{\sqrt[3]{54a^5b^4}}{\sqrt[3]{25c^2}} = \frac{\sqrt[3]{(27a^3b^3)(2a^2b)}}{\sqrt[3]{(5)^2c^2}} = \frac{\sqrt[3]{27a^3b^3}\sqrt[3]{2a^2b}}{\sqrt[3]{(5)^2c^2}} = \frac{3ab\sqrt[3]{2a^2b}}{\sqrt[3]{(5)^2c^2}} = \frac{3ab\sqrt[3]{2a^2b}}{\sqrt[3]{(5)^2c^2}}\left(\frac{\sqrt[3]{(5)^1c^1}}{\sqrt[3]{(5)^1c^1}}\right)$$

$$= \frac{3ab\sqrt[3]{2a^2b}\sqrt[3]{(5)^1c^1}}{5c} = \frac{3ab\sqrt{(2a^2b)(5c)}}{5c} = \frac{3ab\sqrt{10a^2bc}}{5c}$$

70) $\sqrt[3]{\dfrac{27x^6y^3}{2z^2}}$

$$\sqrt[3]{\frac{27x^6y^3}{2z^2}} = \frac{\sqrt[3]{27x^6y^3}}{\sqrt[3]{2z^2}} = \frac{3x^2y}{\sqrt[3]{(2)^1z^2}} = \frac{3x^2y}{\sqrt[3]{(2)^1z^2}}\left(\frac{\sqrt[3]{(2)^2z^1}}{\sqrt[3]{(2)^2z^1}}\right) = \frac{3x^2y\sqrt[3]{(2)^2z^1}}{2z} = \frac{3x^2y\sqrt[3]{4z}}{2z}$$