Note: We will cover up to simple division in this section (examples 1 to 7).

Additional examples

2) $x^{2/5}(x^{3/5}-x^{8/5})$

$$x^{2/5}(x^{3/5}-x^{8/5})=x^{\left(\frac{2}{5}+\frac{3}{5}\right)}-x^{\left(\frac{2}{5}+\frac{8}{5}\right)}=x^{\left(\frac{2+3}{5}\right)}-x^{\left(\frac{2+8}{5}\right)}=x^{\left(\frac{5}{5}\right)}-x^{\left(\frac{10}{5}\right)}=x^1-x^2=x-x^2$$

4) $a^{1/4}(a^{3/4}+a^{7/4})$

$$a^{1/4}(a^{3/4}+a^{7/4})=a^{\left(\frac{1}{4}+\frac{3}{4}\right)}+a^{\left(\frac{1}{4}+\frac{7}{4}\right)}=a^{\left(\frac{1+3}{4}\right)}+a^{\left(\frac{1+7}{4}\right)}=a^{\left(\frac{4}{4}\right)}+a^{\left(\frac{8}{4}\right)}=a^1+a^2=a+a^2$$

6) $5x^{1/2}(4x^{5/2}+3x^{3/2}+2x^{1/2})$

$$5x^{1/2}(4x^{5/2}+3x^{3/2}+2x^{1/2})=5(4)x^{\left(\frac{1}{2}+\frac{5}{2}\right)}+5(3)x^{\left(\frac{1}{2}+\frac{3}{2}\right)}+5(2)x^{\left(\frac{1}{2}+\frac{1}{2}\right)}=20x^{\left(\frac{6}{2}\right)}+15x^{\left(\frac{4}{2}\right)}+10x^{\left(\frac{2}{2}\right)}$$
$$=20x^3+15x^2+10x^1=20x^3+15x^2+10x$$

8) $8x^{4/5}y^{1/3}(4x^{6/5}y^{-1/3}-12x^{-4/5}y^{5/3})$

$$8x^{4/5}y^{1/3}(4x^{6/5}y^{-1/3}-12x^{-4/5}y^{5/3})=8(4)x^{\left(\frac{4}{5}+\frac{6}{5}\right)}y^{\left(\frac{1}{3}+\left(\frac{-1}{3}\right)\right)}-8(12)x^{\left(\frac{4}{5}+\left(\frac{-4}{5}\right)\right)}y^{\left(\frac{1}{3}+\frac{5}{3}\right)}=32x^{\left(\frac{15}{5}\right)}y^0-96x^0y^{\left(\frac{6}{3}\right)}$$
$$=32x^3-96y^2$$

10) $(x^{2/3}-5)(x^{2/3}+2)$

$$(x^{2/3}-5)(x^{2/3}+2)=(x^{2/3})(x^{2/3})+2x^{2/3}-5x^{2/3}-10=x^{4/3}-3x^{2/3}-10$$

12) $(a^{1/2}-6)(a^{1/2}-2)$

$$(a^{1/2}-6)(a^{1/2}-2)=(a^{1/2})(a^{1/2})-2a^{1/2}-6a^{1/2}+12=a^{2/2}-8a^{1/2}+12=a-8a^{1/2}+12$$

14) $(5y^{1/3}-2)(4y^{1/3}+3)$

$$(5y^{1/3}-2)(4y^{1/3}+3)=(5y^{1/3})(4y^{1/3})+3(5y^{1/3})-2(4y^{1/3})-6=20y^{2/3}+15y^{1/3}-8y^{1/3}-6$$
$$=20y^{2/3}+7y^{1/3}-6$$

16) $(4x^{2/3}-2y^{1/2})(5x^{2/3}-3y^{1/2})$

$$(4x^{2/3}-2y^{1/2})(5x^{2/3}-3y^{1/2})=(4x^{2/3})(5x^{2/3})+(4x^{2/3})(-3y^{1/2})+(5x^{2/3})(-2y^{1/2})+(-3y^{1/2})(-2y^{1/2})$$
$$=20x^{4/3}-12x^{2/3}y^{1/2}-10x^{2/3}y^{1/2}+6y^{2/2}=20x^{4/3}-22x^{2/3}y^{1/2}+6y$$

18) $(t^{1/2}-3)^2$

$$(t^{1/2}-3)^2=(t^{1/2}-3)(t^{1/2}-3)=(t^{1/2})(t^{1/2})-3t^{1/2}-3t^{1/2}+9=t^{2/2}-6t^{1/2}+9=t-6t^{1/2}+9$$

20) $(x^{3/2}-6)^2$

$(x^{3/2}-6)^2=(x^{3/2}-6)(x^{3/2}-6)=(x^{3/2})(x^{3/2})-6x^{3/2}-6x^{3/2}+36=x^{6/2}-12x^{3/2}+36=x^3-12x^{3/2}+36$

22) $(a^{1/2}+b^{1/2})^2$

$(a^{1/2}+b^{1/2})^2=(a^{1/2}+b^{1/2})(a^{1/2}+b^{1/2})=(a^{1/2})(a^{1/2})+(a^{1/2})(b^{1/2})+(a^{1/2})(b^{1/2})+(b^{1/2})(b^{1/2})$

$=a^{2/2}+a^{1/2}b^{1/2}+a^{1/2}b^{1/2}+b^{2/2}=a+2a^{1/2}b^{1/2}+b$

24) $(5x^{1/2}+4y^{1/2})^2$

$(5x^{1/2}+4y^{1/2})^2=(5x^{1/2}+4y^{1/2})(5x^{1/2}+4y^{1/2})=(5x^{1/2})(5x^{1/2})+(5x^{1/2})(4y^{1/2})+(5x^{1/2})(4y^{1/2})+(4y^{1/2})(4y^{1/2})$

$=25x^{2/2}+20x^{1/2}y^{1/2}+20x^{1/2}y^{1/2}+16y^{2/2}=25x+40x^{1/2}y^{1/2}+16y$

26) $(a^{1/2}-5^{1/2})(a^{1/2}+5^{1/2})$

$(a^{1/2}-5^{1/2})(a^{1/2}+5^{1/2})=(a^{1/2})(a^{1/2})+(a^{1/2})(5^{1/2})+(a^{1/2})(-5^{1/2})+(-5^{1/2})(5^{1/2})$

$=a^{2/2}+5^{1/2}a^{1/2}-5^{1/2}a^{1/2}-5^{2/2}=a-5$

28) $(x^{5/2}+y^{5/2})(x^{5/2}-y^{5/2})$

$(x^{5/2}+y^{5/2})(x^{5/2}-y^{5/2})=(x^{5/2})(x^{5/2})+(x^{5/2})(-y^{5/2})+(x^{5/2})(y^{5/2})+(y^{5/2})(-y^{5/2})$

$=x^{10/2}-x^{5/2}y^{5/2}+x^{5/2}y^{5/2}-y^{10/2}=x^5-y^5$

30) $(t^{1/2}-5^{3/2})(t^{1/2}+5^{3/2})$

$(t^{1/2}-5^{3/2})(t^{1/2}+5^{3/2})=(t^{1/2})(t^{1/2})+(t^{1/2})(5^{3/2})+(t^{1/2})(-5^{3/2})+(-5^{3/2})(5^{3/2})$

$=t^{2/2}+5^{3/2}t^{1/2}-5^{3/2}t^{1/2}-5^{6/2}=t-5^3=t-125$

32) $(3x^{1/2}+2^{3/2})(3x^{1/2}-2^{3/2})$

$(3x^{1/2}+2^{3/2})(3x^{1/2}-2^{3/2})=(3x^{1/2})(3x^{1/2})+(3x^{1/2})(-2^{3/2})+(3x^{1/2})(2^{3/2})+(2^{3/2})(-2^{3/2})$

$=9x^{2/2}-2^{3/2}x^{1/2}+2^{3/2}x^{1/2}-2^{6/2}=9x-2^3=9x-8$

34) $(x^{1/3}-y^{1/3})(x^{2/3}+x^{2/3}y^{1/3}+y^{2/3})$

$(x^{1/3}-y^{1/3})(x^{2/3}+x^{1/3}y^{1/3}+y^{2/3})$

$=(x^{1/3})(x^{2/3})+(x^{1/3})(x^{1/3}y^{1/3})+(x^{1/3})(y^{2/3})+(-y^{1/3})(y^{2/3})+(-y^{1/3})(x^{1/3}y^{1/3})+(-y^{1/3})(y^{2/3})$

$=x^{3/3}+x^{2/3}y^{1/3}+x^{1/3}y^{2/3}-x^{2/3}y^{1/3}-x^{1/3}y^{2/3}-y^{3/3}=x-y$

36) $(a^{1/3}+3)(a^{2/3}-3a^{1/3}+9)$

$$(a^{1/3}+3)(a^{2/3}-3a^{1/3}+9)=(a^{1/3})(a^{2/3})+(a^{1/3})(-3a^{1/3})+(a^{1/3})(9)+(3)(a^{2/3})+(3)(-3a^{1/3})+(3)(9)$$
$$=a^{3/3}-3a^{2/3}+9a^{1/3}+3a^{2/3}-9a^{1/3}+27=a+27$$

38) $(3x^{1/3}-1)(9x^{2/3}+3x^{1/3}+1)$

$$(3x^{1/3}-1)(9x^{2/3}+3x^{1/3}+1)=(3x^{1/3})(9x^{2/3})+(3x^{1/3})(3x^{1/3})+(3x^{1/3})(1)+(-1)(9x^{2/3})+(-1)(3x^{1/3})+(-1)(1)$$
$$=27x^{3/3}+9x^{2/3}+3x^{1/3}-9x^{2/3}-3x^{1/3}-1=27x-1$$

40) $(t^{1/4}-2)(t^{1/4}+2)(t^{1/2}+4)$

$$(t^{1/4}-2)(t^{1/4}+2)(t^{1/2}+4)=(t^{1/2}+4)\{(t^{1/4}-2)(t^{1/4}+2)\}=(t^{1/2}+4)\{(t^{1/4})(t^{1/4})+2t^{1/4}-2t^{1/4}-4\}$$
$$=(t^{1/2}+4)\{t^{2/4}-4\}=(t^{1/2}+4)\{t^{1/2}-4\}=(t^{1/2})(t^{1/2})-4t^{1/2}+4t^{1/2}-16$$
$$=t^{2/2}-16=t-16$$

42) $\dfrac{25x^{1/4}+30x^{3/4}}{5x^{1/4}}$

$$\frac{25x^{1/4}+30x^{3/4}}{5x^{1/4}}=\frac{25x^{1/4}}{5x^{1/4}}+\frac{30x^{3/4}}{5x^{1/4}}=5+6x^{\left(\left(\frac{3}{4}\right)-\left(\frac{1}{4}\right)\right)}=5+6x^{\left(\frac{2}{4}\right)}=5+6x^{1/2}$$

44) $\dfrac{12x^{4/3}y^{1/3}-18x^{1/3}y^{4/3}}{6x^{1/3}y^{1/3}}$

$$\frac{12x^{4/3}y^{1/3}-18x^{1/3}y^{4/3}}{6x^{1/3}y^{1/3}}=\frac{12x^{4/3}y^{1/3}}{6x^{1/3}y^{1/3}}-\frac{18x^{1/3}y^{4/3}}{6x^{1/3}y^{1/3}}=2x^{\left(\left(\frac{4}{3}\right)-\left(\frac{1}{3}\right)\right)}-3y^{\left(\left(\frac{4}{3}\right)-\left(\frac{1}{3}\right)\right)}=2x^{\left(\frac{3}{3}\right)}-3y^{\left(\frac{3}{3}\right)}=2x-3y$$

46) $\dfrac{24a^{9/5}b^{3/5}-16a^{4/5}b^{8/5}}{8a^{4/5}b^{3/5}}$

$$\frac{24a^{9/5}b^{3/5}-16a^{4/5}b^{8/5}}{8a^{4/5}b^{3/5}}=\frac{24a^{9/5}b^{3/5}}{8a^{4/5}b^{3/5}}-\frac{16a^{4/5}b^{8/5}}{8a^{4/5}b^{3/5}}=3a^{\left(\left(\frac{9}{5}\right)-\left(\frac{4}{5}\right)\right)}-2b^{\left(\left(\frac{8}{5}\right)-\left(\frac{3}{5}\right)\right)}=3a^{\left(\frac{5}{5}\right)}-2b^{\left(\frac{5}{5}\right)}=3a-2b$$