Definition: The **least common denominator** for a set of denominators is the smallest expression that is divisible by each of the denominators.

Additional examples

2) $\frac{5}{6}+\frac{1}{3}$

$$LCD = 6 = (3)(2)$$

$$\frac{5}{6}+\frac{1}{3}=\frac{5+1(2)}{6}=\frac{5+2}{6}=\frac{7}{6}$$

4) $\frac{5}{8}-\frac{1}{4}$

$$LCD = 8 = (4)(2)$$

$$\frac{5}{8}-\frac{1}{4}=\frac{5-1(2)}{8}=\frac{5-2}{8}=\frac{3}{8}$$

6) $\frac{3}{4}+\frac{2}{3}$

$$LCD = 12 = (4)(3)$$

$$\frac{3}{4}+\frac{2}{3}=\frac{3(3)+2(4)}{12}=\frac{9+6}{12}=\frac{15}{12}$$

8) $\frac{6}{28}-\frac{5}{42}$

$$LCD = 42 = (14)(3)$$

$$\frac{6}{28}-\frac{5}{42}=\frac{3(2)}{14(2)}-\frac{5}{42}=\frac{3}{14}-\frac{5}{42}=\frac{3(3)-5}{42}=\frac{9-5}{42}=\frac{4}{42}=\frac{2}{21}$$

10) $\frac{1}{3}-\frac{5}{6}+\frac{5}{12}$

$$LCD = 12 = (6)(2) = (3)(4)$$

$$\frac{1}{3}-\frac{5}{6}+\frac{5}{12}=\frac{1(4)-5(2)+5}{12}=\frac{4-10+5}{12}=\frac{-1}{12}$$

12) $\frac{5x}{5x+2}+\frac{2}{5x+2}$

$$LCD = (5x+2)$$

$$\frac{5x}{5x+2}+\frac{2}{5x+2}=\frac{5x}{(5x+2)}+\frac{2}{(5x+2)}=\frac{5x+2}{(5x+2)}=\frac{(5x+2)}{(5x+2)}=\frac{1}{1}=1$$

14) $\frac{8}{y+8}+\frac{y}{y+8}$

$$LCD=(y+8)$$

$$\frac{8}{y+8}+\frac{y}{y+8}=\frac{8}{(y+8)}+\frac{y}{(y+8)}=\frac{8+y}{(y+8)}=\frac{y+8}{(y+8)}=\frac{(y+8)}{(y+8)}=\frac{1}{1}=1$$

16) $\frac{x}{x^2-y^2}+\frac{y}{x^2-y^2}$

$$LCD=x^2-y^2=(x+y)(x-y)$$

$$\frac{x}{x^2-y^2}+\frac{y}{x^2-y^2}=\frac{x+y}{x^2-y^2}=\frac{(x+y)}{(x+y)(x-y)}=\frac{(1)}{(1)(x-y)}=\frac{1}{(x-y)}=\frac{1}{x-y}$$

18) $\frac{2x-4}{x+2}-\frac{x-6}{x+2}$

$$LCD=(x+2)$$

$$\frac{2x-4}{x+2}-\frac{x-6}{x+2}=\frac{(2x-4)}{(x+2)}-\frac{(x-6)}{(x+2)}=\frac{(2x-4)-(x-6)}{(x+2)}=\frac{2x-4-x+6}{(x+2)}=\frac{x+2}{(x+2)}=\frac{(x+2)}{(x+2)}=\frac{1}{1}=1$$

20) $\frac{3}{a}+\frac{2}{a^2}-\frac{1}{a^3}$

$$LCD=a^3=a(a^2)=a^2(a)$$

$$\frac{3}{a}+\frac{2}{a^2}-\frac{1}{a^3}=\frac{3(a^2)+2(a)-1}{a^3}=\frac{3a^2+2a-1}{a^3}=\frac{(a+1)(3a-1)}{a^3}$$

22) $\frac{7x-1}{3x+2}-\frac{4x-3}{3x+2}$

$$LCD=(3x+2)$$

$$\frac{7x-1}{3x+2}-\frac{4x-3}{3x+2}=\frac{(7x-1)}{(3x+2)}-\frac{(4x-3)}{(3x+2)}=\frac{(7x-1)-(4x-3)}{(3x+2)}=\frac{7x-1-4x+3}{(3x+2)}=\frac{3x+2}{(3x+2)}=\frac{(3x+2)}{(3x+2)}=\frac{1}{1}=1$$

24) $\frac{5}{3t}-\frac{4}{t^2}$

$$LCD=3t^2=3t(t)=t^2(3)$$

$$\frac{5}{3t}-\frac{4}{t^2}=\frac{5(t)-4(3)}{3t^2}=\frac{5t-12}{3t^2}$$

26) $\frac{x+1}{x-2}-\frac{4x+7}{5x-10}$

$$LCD=5(x-2)$$

$$\frac{x+1}{x-2}-\frac{4x+7}{5x-10}=\frac{(x+1)}{(x-2)}-\frac{(4x+7)}{5(x-2)}=\frac{5(x+1)-(4x+7)}{5(x-2)}=\frac{5x+5-4x-7}{5(x-2)}=\frac{x-2}{5(x-2)}=\frac{(x-2)}{5(x-2)}=\frac{(1)}{5(1)}=\frac{1}{5}$$

28) $\dfrac{4x+2}{3x+12}-\dfrac{x-2}{x+4}$

$$LCD = 3(x+4)$$

$$\frac{4x+2}{3x+12}-\frac{x-2}{x+4}=\frac{(4x+2)}{3(x+4)}-\frac{(x-2)}{(x+4)}=\frac{(4x+2)-3(x-2)}{3(x+4)}=\frac{4x+2-3x+6}{3(x+4)}=\frac{x+8}{3(x+4)}$$

30) $\dfrac{x+7}{2x+12}+\dfrac{6}{x^2-36}$

$$LCD = 2(x+6)(x-6)$$

$$\frac{x+7}{2x+12}+\frac{6}{x^2-36}=\frac{(x+7)}{2(x+6)}+\frac{6}{(x+6)(x-6)}=\frac{(x+7)(x-6)+6(2)}{2(x+6)(x-6)}=\frac{(x^2+x-42)+12}{2(x+6)(x-6)}$$

$$=\frac{x^2+x-42+12}{2(x+6)(x-6)}=\frac{x^2+x-30}{2(x+6)(x-6)}=\frac{(x+6)(x-5)}{2(x+6)(x-6)}=\frac{(1)(x-5)}{2(1)(x-6)}=\frac{(x-5)}{2(x-6)}=\frac{x-5}{2(x-6)}$$

32) $\dfrac{1}{a+b}+\dfrac{3ab}{a^3+b^3}$

$$LCD = (a+b)(a^2-ab+b^2)$$

$$\frac{1}{a+b}+\frac{3ab}{a^3+b^3}=\frac{1}{(a+b)}+\frac{3ab}{(a+b)(a^2-ab+b^2)}=\frac{1(a^2-ab+b^2)+3ab}{(a+b)(a^2-ab+b^2)}=\frac{a^2-ab+b^2+3ab}{(a+b)(a^2-ab+b^2)}$$

$$=\frac{a^2+2ab+b^2}{(a+b)(a^2-ab+b^2)}=\frac{(a+b)(a+b)}{(a+b)(a^2-ab+b^2)}=\frac{(1)(a+b)}{(1)(a^2-ab+b^2)}$$

$$=\frac{(a+b)}{(a^2-ab+b^2)}=\frac{a+b}{a^2-ab+b^2}$$

34) $\dfrac{1}{3y-2}-\dfrac{18y}{27y^3-8}$

$$LCD = (3y-2)(9y^2+6y+4)$$

$$\frac{1}{3y-2}-\frac{18y}{27y^3-8}=\frac{1}{(3y-2)}-\frac{18y}{(3y-2)(9y^2+6y+4)}=\frac{1(9y^2+6y+4)-18y}{(3y-2)(9y^2+6y+4)}=\frac{9y^2+6y+4-18y}{(3y-2)(9y^2+6y+4)}$$

$$=\frac{9y^2-12y+4}{(3y-2)(9y^2+6y+4)}=\frac{(3y-2)(3y-2)}{(3y-2)(9y^2+6y+4)}=\frac{(1)(3y-2)}{(1)(9y^2+6y+4)}$$

$$=\frac{(3y-2)}{(9y^2+6y+4)}=\frac{3y-2}{9y^2+6y+4}$$

36) $\dfrac{x}{x^2+4x+4}-\dfrac{2}{2+x}$

$$LCD = (x+2)^2 = (x+2)(x+2)$$

$$\frac{x}{x^2+4x+4}-\frac{2}{2+x}=\frac{x}{(x+2)(x+2)}-\frac{2}{(x+2)}=\frac{x-2(x+2)}{(x+2)(x+2)}=\frac{x-2x-4}{(x+2)(x+2)}=\frac{-x-4}{(x+2)(x+2)}=\frac{-x-4}{(x+2)^2}$$

38) $\dfrac{3}{2t-5}+\dfrac{21}{8t^2-14t-15}$

$$LCD=(4t+3)(2t-5)$$

$$\frac{3}{2t-5}+\frac{21}{8t^2-14t-15}=\frac{3}{(2t-5)}+\frac{21}{(4t+3)(2t-5)}=\frac{3(4t+3)+21}{(4t+3)(2t-5)}=\frac{12t+9+21}{(4t+3)(2t-5)}$$

$$=\frac{12t+30}{(4t+3)(2t-5)}=\frac{6(2t+5)}{(4t+3)(2t-5)}$$

40) $\dfrac{-3}{a^2+a-2}+\dfrac{5}{a^2-a-6}$

$$LCD=(a+2)(a-1)(a-3)$$

$$\frac{-3}{a^2+a-2}+\frac{5}{a^2-a-6}=\frac{-3}{(a+2)(a-1)}+\frac{5}{(a+2)(a-3)}=\frac{-3(a-3)+5(a-1)}{(a+2)(a-1)(a-3)}$$

$$=\frac{-3a+9+5a-5}{(a+2)(a-1)(a-3)}=\frac{2a+4}{(a+2)(a-1)(a-3)}$$

$$=\frac{2(a+2)}{(a+2)(a-1)(a-3)}=\frac{2(1)}{(1)(a-1)(a-3)}=\frac{2}{(a-1)(a-3)}$$

42) $\dfrac{1}{27x^3-1}-\dfrac{1}{9x^2-1}$

$$LCD=(3x+1)(3x-1)(9x^2+3x+1)$$

$$\frac{1}{27x^3-1}-\frac{1}{9x^2-1}=\frac{1}{(3x-1)(9x^2+3x+1)}-\frac{1}{(3x+1)(3x-1)}=\frac{1(3x+1)-1(9x^2+3x+1)}{(3x+1)(3x-1)(9x^2+3x+1)}$$

$$=\frac{3x+1-9x^2-3x-1}{(3x+1)(3x-1)(9x^2+3x+1)}=\frac{-9x^2}{(3x+1)(3x-1)(9x^2+3x+1)}$$

44) $\dfrac{9}{9x^2+6x-8}-\dfrac{6}{9x^2-4}$

$$LCD=(3x+4)(3x+2)(3x-2)$$

$$\frac{9}{9x^2+6x-8}-\frac{6}{9x^2-4}=\frac{9}{(3x+4)(3x-2)}-\frac{6}{(3x+2)(3x-2)}=\frac{9(3x+2)-6(3x+4)}{(3x+4)(3x+2)(3x-2)}$$

$$=\frac{27x+18-18x-24}{(3x+4)(3x+2)(3x-2)}=\frac{9x-6}{(3x+4)(3x+2)(3x-2)}$$

$$=\frac{3(3x-2)}{(3x+4)(3x+2)(3x-2)}=\frac{3(1)}{(3x+4)(3x+2)(1)}$$

$$=\frac{3}{(3x+4)(3x+2)}$$

46) $\dfrac{3a}{a^2+7a+10}-\dfrac{2a}{a^2+6a+8}$

$$LCD=(a+5)(a+4)(a+2)$$

$$\frac{3a}{a^2+7a+10}-\frac{2a}{a^2+6a+8}=\frac{3a}{(a+5)(a+2)}-\frac{2a}{(a+4)(a+2)}=\frac{3a(a+4)-2a(a+5)}{(a+5)(a+4)(a+2)}=\frac{3a^2+12a-2a^2-10a}{(a+5)(a+4)(a+2)}$$

$$=\frac{a^2+2a}{(a+5)(a+4)(a+2)}=\frac{a(a+2)}{(a+5)(a+4)(a+2)}=\frac{a(1)}{(a+5)(a+4)(1)}=\frac{a}{(a+5)(a+4)}$$

48) $\dfrac{4x+1}{x^2+5x+4}-\dfrac{x+3}{x^2+4x+3}$

method 1:

$$LCD=(x+4)(x+3)(x+1)$$

$$\frac{4x+1}{x^2+5x+4}-\frac{x+3}{x^2+4x+3}=\frac{(4x+1)}{(x+4)(x+1)}-\frac{(x+3)}{(x+3)(x+1)}=\frac{(4x+1)(x+3)-(x+3)(x+4)}{(x+4)(x+3)(x+1)}$$

$$=\frac{(4x^2+13x+3)-(x^2+7x+12)}{(x+4)(x+3)(x+1)}=\frac{4x^2+13x+3-x^2-7x-12}{(x+4)(x+3)(x+1)}$$

$$=\frac{3x^2+6x-9}{(x+4)(x+3)(x+1)}=\frac{3(x^2+2x-3)}{(x+4)(x+3)(x+1)}=\frac{3(x+3)(x-1)}{(x+4)(x+3)(x+1)}$$

$$=\frac{3(1)(x-1)}{(x+4)(1)(x+1)}=\frac{3(x-1)}{(x+4)(x+1)}$$

method 2:

$$LCD=(x+4)(x+1)$$

$$\frac{4x+1}{x^2+5x+4}-\frac{x+3}{x^2+4x+3}=\frac{(4x+1)}{(x+4)(x+1)}-\frac{(x+3)}{(x+3)(x+1)}=\frac{(4x+1)}{(x+4)(x+1)}-\frac{(1)}{(1)(x+1)}$$

$$=\frac{(4x+1)}{(x+4)(x+1)}-\frac{1}{(x+1)}=\frac{(4x+1)-1(x+4)}{(x+4)(x+1)}=\frac{4x+1-x-4}{(x+4)(x+1)}$$

$$=\frac{3x-3}{(x+4)(x+1)}=\frac{3(x-1)}{(x+4)(x+1)}$$

50) $\dfrac{5x+3}{2x^2+5x+3}-\dfrac{3x+9}{2x^2+7x+6}$

$$LCD=(2x+3)(x+2)(x+1)$$

$$\frac{5x+3}{2x^2+5x+3}-\frac{3x+9}{2x^2+7x+6}=\frac{(5x+3)}{(2x+3)(x+1)}-\frac{(3x+9)}{(2x+3)(x+2)}=\frac{(5x+3)(x+2)-(3x+9)(x+1)}{(2x+3)(x+2)(x+1)}$$

$$=\frac{(5x^2+13x+6)-(3x^2+12x+9)}{(2x+3)(x+2)(x+1)}=\frac{5x^2+13x+6-3x^2-12x-9}{(2x+3)(x+2)(x+1)}$$

$$=\frac{2x^2+x-3}{(2x+3)(x+2)(x+1)}=\frac{(2x+3)(x-1)}{(2x+3)(x+2)(x+1)}=\frac{(1)(x-1)}{(1)(x+2)(x+1)}$$

$$=\frac{(x-1)}{(x+2)(x+1)}=\frac{x-1}{(x+2)(x+1)}$$

52) $\dfrac{-5}{x^2+3x-4}+\dfrac{5}{x^2+2x-3}+\dfrac{1}{x^2+7x+12}$

$$LCD=(x+4)(x+3)(x-1)$$

$$\begin{aligned}\frac{-5}{x^2+3x-4}+\frac{5}{x^2+2x-3}+\frac{1}{x^2+7x+12}&=\frac{-5}{(x+4)(x-1)}+\frac{5}{(x+3)(x-1)}+\frac{1}{(x+4)(x+3)}\\&=\frac{-5(x+3)+5(x+4)+1(x-1)}{(x+4)(x+3)(x-1)}=\frac{-5x-15+5x+20+x-1}{(x+4)(x+3)(x-1)}\\&=\frac{x+4}{(x+4)(x+3)(x-1)}=\frac{(x+4)}{(x+4)(x+3)(x-1)}\\&=\frac{(1)}{(1)(x+3)(x-1)}=\frac{1}{(x+3)(x-1)}\end{aligned}$$

54) $\dfrac{2x+11}{x^2+9x+20}-\dfrac{x+1}{x^2+7x+12}-\dfrac{x+6}{x^2+8x+15}$

$$LCD=(x+5)(x+4)(x+3)$$

$$\begin{aligned}\frac{2x+11}{x^2+9x+20}-\frac{x+1}{x^2+7x+12}-\frac{x+6}{x^2+8x+15}&=\frac{(2x+11)}{(x+5)(x+4)}-\frac{(x+1)}{(x+4)(x+3)}-\frac{(x+6)}{(x+5)(x+3)}\\&=\frac{(2x+11)(x+3)-(x+1)(x+5)-(x+6)(x+4)}{(x+5)(x+4)(x+3)}\\&=\frac{(2x^2+17x+33)-(x^2+6x+5)-(x^2+10x+24)}{(x+5)(x+4)(x+3)}\\&=\frac{2x^2+17x+33-x^2-6x-5-x^2-10x-24}{(x+5)(x+4)(x+3)}\\&=\frac{x+4}{(x+5)(x+4)(x+3)}=\frac{(x+4)}{(x+5)(x+4)(x+3)}\\&=\frac{(1)}{(x+5)(1)(x+3)}=\frac{1}{(x+5)(x+3)}\end{aligned}$$

56) $3-\dfrac{2}{2x+3}$

$$LCD=(2x+3)$$

$$3-\frac{2}{2x+3}=\frac{3}{1}-\frac{2}{(2x+3)}=\frac{3(2x+3)-2}{(2x+3)}=\frac{6x+9-2}{(2x+3)}=\frac{6x+7}{(2x+3)}=\frac{6x+7}{2x+3}$$

58) $7+\dfrac{3}{5-t}$

$$LCD=(5-t)$$

$$7+\frac{3}{5-t}=\frac{7}{1}+\frac{3}{(5-t)}=\frac{7(5-t)+3}{(5-t)}=\frac{35-7t+3}{(5-t)}=\frac{38-7t}{(5-t)}=\frac{38-7t}{5-t}$$

60) $x-\frac{5}{3x+4}+1$

$$LCD=(3x+4)$$

$$x-\frac{5}{3x+4}+1=\frac{x}{1}-\frac{5}{(3x+4)}+\frac{1}{1}=\frac{x(3x+4)-5+1(3x+4)}{(3x+4)}=\frac{3x^2+4x-5+3x+4}{(3x+4)}$$

$$=\frac{3x^2+7x-1}{(3x+4)}=\frac{3x^2+7x-1}{3x+4}$$

62) $\frac{x}{x+3}+\frac{7}{3x+9}-\frac{2}{x^2+3x}$

$$LCD=3x(x+3)=(3)(x)(x+3)$$

$$\frac{x}{x+3}+\frac{7}{3x+9}-\frac{2}{x^2+3x}=\frac{x}{(x+3)}+\frac{7}{3(x+3)}-\frac{2}{x(x+3)}=\frac{x(3)(x)+7(x)-2(3)}{3x(x+3)}=\frac{3x^2+7x-6}{3x(x+3)}$$

$$=\frac{(x+3)(3x-2)}{3x(x+3)}=\frac{(1)(3x-2)}{3x(1)}=\frac{(3x-2)}{3x}=\frac{3x-2}{3x}$$

64) $\frac{1}{x}+\frac{x}{3x+9}-\frac{3}{x^2+3x}$

$$LCD=3x(x+3)=(3)(x)(x+3)$$

$$\frac{1}{x}+\frac{x}{3x+9}-\frac{3}{x^2+3x}=\frac{1}{x}+\frac{x}{3(x+3)}-\frac{3}{x(x+3)}=\frac{1(3)(x+3)+x(x)-3(3)}{3x(x+3)}=\frac{3x+9+x^2-9}{3x(x+3)}=\frac{x^2+3x}{3x(x+3)}$$

$$=\frac{x(x+3)}{3x(x+3)}=\frac{(x)(x+3)}{3(x)(x+3)}=\frac{(1)(1)}{3(1)(1)}=\frac{1}{3}$$