Definition: The **solution set** for an equation is the set of all numbers that, when used in place of the variable, make the equation a true statement.

Definition: Two or more equations with the same solution set are called **equivalent equations**.

Addition Property of Equality:
For any three algebraic expressions *A*, *B*, and *C*,

if $A = B$

then $A + C = B + C$

Multiplication Property of Equality:
For any three algebraic expressions *A*, *B*, and *C*, where $C \neq 0$,

if $A = B$

then $AC = BC$

Definition: Any equation that can be written in the form

$$ax^2 + bx + c = 0$$

where *a*, *b*, and *c* are constants and *a* is not 0 ($a \neq 0$), is called a **quadratic equation**. The form $ax^2 + bx + c = 0$ is called **standard form** for quadratic equations.

Note: The statement above is true for all polynomials with degree 2 and above.

Zero-Factor Property of Equality:
For all real numbers *r* and *s*,

$r \cdot s = 0$ if and only if $r = 0$ or $s = 0$ (or both)

Note: The statement above is true for statements with 3 real numbers and more.

Read **Strategy for Solving Linear Equations in One Variable** on Page 105 and **To Solve an Equation by Factoring** on Page 109 for guidance.

Additional examples
:

2)
$$3x - 5 = 4$$
$$3x - 5 = 4$$
$$3x = -9$$
$$x = -3$$

4)
$$-8 - 5x = -6$$
$$-8 - 5x = -6$$
$$-8 = 5x - 6$$
$$-2 = 5x$$
$$\frac{-2}{5} = x$$

6)
$$-20y + 80 = 30$$
$$-20y + 80 = 30$$
$$80 = 20y + 30$$
$$50 = 20y$$
$$\frac{50}{20} = y$$
$$\frac{5}{2} = y$$

8) $-\frac{5}{3}a+3=23$

$$LCD=3$$
$$-\frac{5}{3}a+3=23$$
$$\left(\frac{3}{1}\right)\left(\frac{-5a}{3}+\frac{3}{1}\right)=\left(\frac{23}{1}\right)\left(\frac{3}{1}\right)$$
$$-5a+9=69$$
$$9=5a+69$$
$$60=5a$$
$$\frac{60}{5}=a$$
$$12=a$$

10) $-x=\frac{1}{2}$

$$LCD=2$$
$$-x=\frac{1}{2}$$
$$\left(\frac{2}{1}\right)\left(\frac{-x}{1}\right)=\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)$$
$$-2x=1$$
$$x=\frac{1}{-2}$$
$$x=\frac{-1}{2}$$

12) $-a=-5$

$$-a=-5$$
$$(-1)(-a)=(-5)(-1)$$
$$a=5$$

14) $8y-2=6y-10$

$$8y-2=6y-10$$
$$2y-2=-10$$
$$2y=-8$$
$$y=-4$$

16) $6(y-3)-5(y+2)=8$

$$6(y-3)-5(y+2)=8$$
$$6y-18-5y-10=8$$
$$y-28=8$$
$$y=36$$

18) $x^2=36$

$$x^2=36$$
$$x^2-36=0$$
$$(x+2)(x-2)=0$$
$$x+2=0 \qquad x-2=0$$
$$x=-2 \qquad x=2$$

20) $20=8-5(2x-3)+4x$

$$20=8-5(2x-3)+4x$$
$$20=8-10x+15+4x$$
$$20=23-6x$$
$$6x+20=23$$
$$6x=3$$
$$x=\frac{3}{6}$$
$$x=\frac{1}{2}$$

22) $\frac{2}{3}x-\frac{3}{4}=\frac{1}{6}x+\frac{21}{4}$

$$LCD=12$$
$$\frac{2}{3}x-\frac{3}{4}=\frac{1}{6}x+\frac{21}{4}$$
$$\left(\frac{12}{1}\right)\left(\frac{2x}{3}-\frac{3}{4}\right)=\left(\frac{x}{6}+\frac{21}{4}\right)\left(\frac{12}{1}\right)$$
$$\left(\frac{12}{1}\right)\left(\frac{2x}{3}\right)-\left(\frac{12}{1}\right)\left(\frac{3}{4}\right)=\left(\frac{x}{6}\right)\left(\frac{12}{1}\right)+\left(\frac{21}{4}\right)\left(\frac{12}{1}\right)$$
$$8x-9=2x+63$$
$$6x-9=63$$
$$6x=72$$
$$x=12$$

24) $x^2+5x-6=0$

$$x^2+5x-6=0$$
$$(x+6)(x-1)=0$$
$$x+6=0 \qquad x-1=0$$
$$x=-6 \qquad x=1$$

26) $x^3+5x^2+6x=0$

$$x^3+5x^2+6x=0$$
$$x(x^2+5x+6)=0$$
$$x(x+3)(x+2)=0$$
$$(x+3)(x+2)(x)=0$$
$$x+3=0 \qquad x+2=0 \qquad x=0$$
$$x=-3 \qquad x=-2$$

28) $90x^2+60x-80=0$

$$90x^2+60x-80=0$$
$$10(9x^2+6x-8)=0$$
$$10(3x+4)(3x-2)=0$$
$$3x+4=0 \qquad 3x-2=0$$
$$3x=-4 \qquad 3x=2$$
$$x=\frac{-4}{3} \qquad x=\frac{2}{3}$$

30) $\frac{1}{2}y^2+\frac{5}{3}=\frac{17}{6}y$

$$LCD=6$$
$$\frac{1}{2}y^2+\frac{5}{3}=\frac{17}{6}y$$
$$\left(\frac{6}{1}\right)\left(\frac{y^2}{2}+\frac{5}{3}\right)=\left(\frac{17y}{6}\right)\left(\frac{6}{1}\right)$$
$$\left(\frac{6}{1}\right)\left(\frac{y^2}{2}\right)+\left(\frac{6}{1}\right)\left(\frac{5}{3}\right)=\left(\frac{17y}{6}\right)\left(\frac{6}{1}\right)$$
$$3y^2+10=17y$$
$$3y^2-17y+10=0$$
$$(3y-2)(y-5)=0$$
$$3y-2=0 \qquad y-5=0$$
$$3y=2 \qquad y=5$$
$$y=\frac{2}{3}$$

32) $800x=100x^2$

$$800x=100x^2$$
$$0=100x^2-800x$$
$$0=100x(x-8)$$
$$100x=0 \qquad x-8=0$$
$$x=0 \qquad x=8$$

34) $(x-7)(x+5)=-20$

$$(x-7)(x+5)=-20$$
$$x^2+5x-7x-35=-20$$
$$x^2-2x-35=-20$$
$$x^2-2x-15=0$$
$$(x+3)(x-5)=0$$
$$x+3=0 \qquad x-5=0$$
$$x=-3 \qquad x=5$$

36) $(x+2)^2=9x$

$$(x+2)^2=9x$$
$$(x+2)(x+2)=9x$$
$$x^2+2x+2x+4=9x$$
$$x^2+4x+4=9x$$
$$x^2-5x+4=0$$
$$(x-1)(x-4)=0$$
$$x-1=0 \qquad x-4=0$$
$$x=1 \qquad x=4$$

38) $x^3+5x^2-4x-20=0$

$$x^3+5x^2-4x-20=0$$
$$(x^3+5x^2)-(4x+20)=0$$
$$x^2(x+5)-4(x+5)=0$$
$$\{x^2(x+5)-4(x+5)\}=0$$
$$(x+5)\{x^2-4\}=0$$
$$(x+5)\{(x+2)(x-2)\}=0$$
$$(x+5)(x+2)(x-2)=0$$
$$x+5=0 \qquad x+2=0 \qquad x-2=0$$
$$x=-5 \qquad x=-2 \qquad x=2$$

40) $7-3x=8x-4$

$$7-3x=8x-4$$
$$7=11x-4$$
$$11=11x$$
$$1=x$$

42) $\frac{2}{7}t^2-\frac{7}{2}=0$

$$LCD=14$$
$$\frac{2}{7}t^2-\frac{7}{2}=0$$
$$\left(\frac{14}{1}\right)\left(\frac{2t^2}{7}-\frac{7}{2}\right)=\left(\frac{0}{1}\right)\left(\frac{14}{1}\right)$$
$$\left(\frac{14}{1}\right)\left(\frac{2t^2}{7}\right)-\left(\frac{14}{1}\right)\left(\frac{7}{2}\right)=\left(\frac{0}{1}\right)\left(\frac{14}{1}\right)$$
$$4t^2-49=0$$
$$(2t)^2-(7)^2=0$$
$$(2t+7)(2t-7)=0$$
$$2t+7=0 \qquad 2t-7=0$$
$$2t=-7 \qquad 2t=7$$
$$t=\frac{-7}{2} \qquad t=\frac{7}{2}$$

44) $5+2(4x-4)=3(2x-1)$

$$5+2(4x-4)=3(2x-1)$$
$$5+8x-8=6x-3$$
$$8x-3=6x-3$$
$$2x-3=-3$$
$$2x=0$$
$$x=0$$

46) $-\frac{1}{6}x+\frac{2}{3}=\frac{1}{4}$

$$LCD=12$$
$$-\frac{1}{6}x+\frac{2}{3}=\frac{1}{4}$$
$$\left(\frac{12}{1}\right)\left(\frac{-x}{6}+\frac{2}{3}\right)=\left(\frac{1}{4}\right)\left(\frac{12}{1}\right)$$
$$\left(\frac{12}{1}\right)\left(\frac{-x}{6}\right)+\left(\frac{12}{1}\right)\left(\frac{2}{3}\right)=\left(\frac{1}{4}\right)\left(\frac{12}{1}\right)$$
$$-2x+8=3$$
$$8=2x+3$$
$$5=2x$$
$$\frac{5}{2}=x$$

48) $\frac{1}{3}x+\frac{1}{4}x+\frac{1}{5}x=\frac{47}{60}$

$$LCD=60$$
$$\frac{1}{3}x+\frac{1}{4}x+\frac{1}{5}x=\frac{47}{60}$$
$$\left(\frac{60}{1}\right)\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{5}\right)=\left(\frac{47}{60}\right)\left(\frac{60}{1}\right)$$
$$\left(\frac{60}{1}\right)\left(\frac{x}{3}\right)+\left(\frac{60}{1}\right)\left(\frac{x}{4}\right)+\left(\frac{60}{1}\right)\left(\frac{x}{5}\right)=\left(\frac{47}{60}\right)\left(\frac{60}{1}\right)$$
$$20x+15x+12x=47$$
$$47x=47$$
$$x=1$$

50) $(3r+2)(r-1)=-(7r-7)$

$$(3r+2)(r-1)=-(7r-7)$$
$$3r^2-3r+2r-2=-7r+7$$
$$3r^2-r-2=-7r+7$$
$$3r^2+6r-9=0$$
$$3(r^2+2r-3)=0$$
$$3(r+3)(r-1)=0$$
$$r+3=0 \qquad r-1=0$$
$$r=-3 \qquad r=1$$

52) $16a^3 = 25a$

$$16a^3 = 25a$$
$$16a^3 - 25a = 0$$
$$a\{16a^2 - 25\} = 0$$
$$a\{(4a)^2 - (5)^2\} = 0$$
$$a\{(4a+5)(4a-5)\} = 0$$
$$(4a+5)(a)(4a-5) = 0$$

$4a+5=0$ $\quad$ $4a-5=0$

$4a=-5$ $\quad$ $a=0$ $\quad$ $4a=5$

$a=\frac{-5}{4}$ $\quad$ $a=\frac{5}{4}$

54) $9x^3 + 18x^2 - 4x - 8 = 0$

$$9x^3 + 18x^2 - 4x - 8 = 0$$
$$(9x^3 + 18x^2) - (4x+8) = 0$$
$$9x^2(x+2) - 4(x+2) = 0$$
$$\{9x^2(x+2) - 4(x+2)\} = 0$$
$$(x+2)\{9x^2 - 4\} = 0$$
$$(x+2)\{(3x)^2 - (2)\} = 0$$
$$(x+2)\{(3x+2)(3x-2)\} = 0$$
$$(x+2)(3x+2)(3x-2) = 0$$

$x+2=0$ $\quad$ $3x+2=0$ $\quad$ $3x-2=0$

$x=-2$ $\quad$ $3x=-2$ $\quad$ $3x=2$

$x=\frac{-2}{3}$ $\quad$ $x=\frac{2}{3}$

56) $-\frac{1}{15}x^2 = \frac{2}{5}x$

$$LCD = 15$$
$$-\frac{1}{15}x^2 = \frac{2}{5}x$$
$$\left(\frac{15}{1}\right)\left(\frac{-x^2}{15}\right) = \left(\frac{2x}{5}\right)\left(\frac{15}{1}\right)$$
$$-x^2 = 6x$$
$$0 = x^2 + 6x$$
$$0 = (x+6)x$$

$x+6=0$ $\quad$ $x=0$

$x=-6$

58) $0.02x + 0.45 = 0.08x^2$

Since the maximum number of decimal places is 2, we can multiply both sides by 100 so that we can remove the decimal numbers.

$$0.02x + 0.45 = 0.08x^2$$
$$(100)(0.02x + 0.45) = (0.08x^2)(100)$$
$$(100)(0.02x) + (100)(0.45) = (0.08x^2)(100)$$
$$2x + 45 = 8x^2$$
$$0 = 8x^2 - 2x - 45$$
$$0 = (4x+9)(2x-5)$$

$4x+9=0$ $\quad$ $x-5=0$

$4x=-9$ $\quad$ $2x=5$

$x=\frac{-9}{4}$ $\quad$ $x=\frac{5}{2}$

60) $12x^3 = 48x^5$

$$12x^3 = 48x^5$$
$$0 = 48x^5 - 12x^3$$
$$0 = 12x^3\{4x^2 - 1\}$$
$$0 = 12x^3\{(2x)^2 - (1)^2\}$$
$$0 = 12x^3\{(2x+1)(2x-1)\}$$
$$0 = (2x+1)(12x^3)(2x-1)$$

$2x+1=0$ $\quad$ $12x^3=0$ $\quad$ $2x-1=0$

$2x=-1$ $\quad$ $x^3=0$ $\quad$ $2x=1$

$x=\frac{-1}{2}$ $\quad$ $x=0$ $\quad$ $x=\frac{1}{2}$

62) $0.065x + 0.05(15000 - x) = 870$

$$0.065x + 0.05(15000 - x) = 870$$
$$0.065x + 750 - 0.05x = 870$$
$$0.065x + 750 - 0.050x = 870$$
$$0.015x + 750 = 870$$
$$0.015x = 120$$
$$(1000)(0.015x) = (120)(1000)$$
$$15x = (120)(1000)$$
$$x = \frac{(120)(1000)}{15}$$
$$x = 8000$$

64) $\frac{1}{2}x^2 - \frac{1}{18} = 0$

$$LCD = 18$$
$$\frac{1}{2}x^2 - \frac{1}{18} = 0$$
$$\left(\frac{18}{1}\right)\left(\frac{x^2}{2} - \frac{1}{18}\right) = \left(\frac{0}{1}\right)\left(\frac{18}{1}\right)$$
$$\left(\frac{18}{1}\right)\left(\frac{x^2}{2}\right) - \left(\frac{18}{1}\right)\left(\frac{1}{18}\right) = \left(\frac{0}{1}\right)\left(\frac{18}{1}\right)$$
$$9x^2 - 1 = 0$$
$$(3x)^2 - (1)^2 = 0$$
$$(3x+1)(3x-1) = 0$$
$$3x+1 = 0 \qquad 3x-1 = 0$$
$$3x = -1 \qquad 3x = 1$$
$$x = \frac{-1}{3} \qquad x = \frac{1}{3}$$

66) $3x^3 - x^2 - 12x + 4 = 0$

$$3x^3 - x^2 - 12x + 4 = 0$$
$$(3x^3 - x^2) - (12x - 4) = 0$$
$$x^2(3x-1) - 4(3x-1) = 0$$
$$\{x^2(3x-1) - 4(3x-1)\} = 0$$
$$(3x-1)\{x^2 - 4\} = 0$$
$$(3x-1)\{x^2 - (2)^2\} = 0$$
$$(3x-1)\{(x+2)(x-2)\} = 0$$
$$(x+2)(3x-1)(x-2) = 0$$
$$x+2 = 0 \qquad 3x-1 = 0 \qquad x-2 = 0$$
$$x = -2 \qquad 3x = 1 \qquad x = 2$$
$$x = \frac{1}{3}$$

68) $7x - 14 = 7(x-2)$

$$7x - 14 = 7(x-2)$$
$$7x - 14 = 7x - 14$$
$$-14 = -14$$

This is a true statement, so the answer is all real numbers.

70) $7y + 5 - 2y - 3 = 6 + 5y - 4$

$$7y + 5 - 2y - 3 = 6 + 5y - 4$$
$$5y + 2 = 2 + 5y$$
$$2 = 2$$

This is a true statement, so the answer is all real numbers.

72) $5(2t-1) + 1 = 2t - 4 + 8t$

$$5(2t-1) + 1 = 2t - 4 + 8t$$
$$10t - 5 + 1 = 10t - 4$$
$$10t - 4 = 10t - 4$$
$$-4 = -4$$

This is a true statement, so the answer is all real numbers.

74) $5x - 2(x+3) = -3(2-x)$

$$5x - 2(x+3) = -3(2-x)$$
$$5x - 2x - 6 = -6 + 3x$$
$$3x - 6 = -6 + 3x$$
$$-6 = -6$$

This is a true statement, so the answer is all real numbers.

76) $3(2x-5) - 4(x-3) = -2(4-x)$

$$3(2x-5) - 4(x-3) = -2(4-x)$$
$$6x - 15 - 4x + 12 = -8 + 2x$$
$$2x - 3 = -8 + 2x$$
$$-3 \neq -8$$

This is a false statement, so there is no solution.