## Parabola:

The general equations of Parabola:

1) Opening up or down: $(x-h)^{2}=4 p(y-k)$ where the vertex of the parabola is $(h, k)$; focus: $(h, k+p)$; directrix: $y=k-p$.

2) Opening left or right: $(y-k)^{2}=4 p(x-h)$ where the vertex of the parabola is $(h, k)$; focus: $(h+p, k)$; directrix: $x=h-p$.
When $p>0$ :

## Ellipse:

The general equations of Ellipse:

1) Longer horizontally:
$\frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}=1$
where center: $(h, k)$,
vertices: $(h \pm a, k)$,
foci: $\begin{gathered}c^{2}=a^{2}-b^{2} \\ (h \pm c, k)\end{gathered}$
Length of Major Axis: $2 a \leftrightarrow$ length of minor axis: $2 b \downarrow$

2) Longer vertically:
$\frac{(y-k)^{2}}{a^{2}}+\frac{(x-h)^{2}}{b^{2}}=1$
where center: $(h, k)$,
vertices: $(h, k \pm a)$,
foci: $\begin{gathered}c^{2}=a^{2}-b^{2} \\ (h, k \pm c)\end{gathered}$
Length of Major Axis: $2 a \downarrow$
length of minor axis: $2 b \leftrightarrow$
$<$ minor axis: $2 b \longrightarrow$


Major Axis: $2 a$

## Hyperbola:

The general equations of Hyperbola:

1) Opening sideways: $\quad \frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1 \quad$ where center: $(h, k)$, vertices: $(h \pm a, k)$, foci: $\begin{gathered}c^{2}=a^{2}+b^{2} \\ (h \pm c, k)\end{gathered} \quad$ Asymptotes: $m= \pm \frac{b}{a} \quad y-k=m(x-h) \Rightarrow y-k= \pm \frac{b}{a}(x-h)$ Length of Transverse Axis: $2 a \leftrightarrow$ length of conjugate axis: $2 b \uparrow$

2) Opening up and down: $\quad \frac{(y-k)^{2}}{a^{2}}-\frac{(x-h)^{2}}{b^{2}}=1 \quad$ where center: $(h, k)$, vertices: $(h, k \pm a)$, foci: $\begin{gathered}c^{2}=a^{2}+b^{2} \\ (h, k \pm c)\end{gathered} \quad$ Asymptotes: $m= \pm \frac{a}{b} \quad y-k=m(x-h) \Rightarrow y-k= \pm \frac{a}{b}(x-h)$
Length of Transverse Axis: $2 a \downarrow$ length of conjugate axis: $2 b \leftrightarrow$


A typical sketching conic section exercises will have equations given in the form shown below:

$$
A x^{2}+C y^{2}+D x+E y+F=0
$$

You will need to use the complete the square technique so that your result will be in one of the forms shown on previous pages.

If only one of the variables $x$ or $y$ is squared (this means either $A=0$ or $C=0$ ), then the graph is a possible Parabola.

If both coefficients $A$ and $C$ have same sign (this means $A>0, \quad C>0$ or $A<0, C<0$ ), then the graph is a possible Ellipse.

If the coefficients $A$ and $C$ have different signs (this means $A>0, \quad C<0$ or $A<0, \quad C>0$ ), then the graph is a possible Hyperbola.

The reason I'm mentioning that we have a possible conic section because we may have a empty graph.
For example, in case of a possible ellipse; we may end up after completing the squares an expression below:

$$
\frac{(y-k)^{2}}{a^{2}}+\frac{(x-h)^{2}}{b^{2}}=-1
$$

When this happens, we don't have a conic section graph because the graph does not exist.

## Rotation of Axes:

Sometimes we may need to find and sketch a graph of a conic section where graph is rotated some angle pivoted on the origin.

For example: a parabola opening diagonally (see figure to the right)

The equation of conic section where there is a rotation involved are given in form below:

$$
A x^{2}+B x y+C y^{2}+D x+E y+F=0
$$

There is a new term that we have not seen before (the $x y$ term).
When the $x y$ term appears in the conic equation, DO NOT draw a conclusion of which possible conic section graphs you will get (method shown in previous page). Use the method after converting the equation in the form of new axes, not at the beginning.


To eliminate the $x y$-term in the general quadratic equation

$$
A x^{2}+B x y+C y^{2}+D x+E y+F=0 \quad \text { where } \quad B \neq 0
$$

rotate the coordinate axes through an angle $\theta$ that satisfies

$$
\cot (2 \theta)=\frac{A-C}{B}
$$

Then use the conversion formulas:

$$
x=\hat{x} \cos \theta-\hat{y} \sin \theta \text { and } y=\hat{x} \sin \theta+\hat{y} \cos \theta
$$

This will eliminate the $x y$-term and convert into the form given on top of page 4 in terms of new variables $\hat{x}$ and $\hat{y}$.

