Math 391 — **EXAM #1** — October 7, 2013

1. (10 pts) For each of the following differential equations, state whether it is **linear or nonlinear** (justify your answer) and determine the **order**:

   (a) $y''' + y^3 y' = x$

   (b) $\sin(x)y'' + x^2 y = 0$

   (c) $y' = \sin(xy)$

   (d) $\frac{dy}{dt} = y^6$

   (e) $u_{xx} + u_{yy} + u_{zz} = 0$

2. (25 pts.) Find the solution of the initial value problem $y' + 2y = te^{-2t},\ y(1) = 0$:

3. (20 pts.) Solve $(ye^{-x} - xye^{-x})dx + (xe^{-x} + e^y)dy = 0$:

4. (25 pts) Solve $(x^2 + 3xy + y^2)dx - x^2 dy = 0$:

5. A tank with a capacity of 100 gallons initially contains 50 gallons of water with 10 pounds of salt in solution. Brine with a concentration of 7 pounds per gallon is pumped in at a rate of 3 gallons per minute and a well-stirred mixture is pumped out at a rate of 1 gallon per minute.

   (a) (5 pts.) Compute the time $t^\star$ when the tank is filled:

   (b) (15 pts.) Set-up **(but DO NOT solve)** an initial value problem (equation plus initial conditions) to determine the quantity $Q(t)$ of the amount of salt (in pounds) in the tank t minutes after the pumping begins, where $t < t^\star$: