Math 324 — **Exam #1** — April 7, 2003

Please PRINT your name and ID# on the cover of each exam book you use. Write clearly and cross-out work not to be graded. Make your arguments rigorous and brief. Partial credit will be given. Total: 100 pts.

1. Give **definitions** of ANY FOUR OF the following FIVE (boldface) words. (20 pts.)

   (a) The **upper Darboux sum** of a bounded function f with respect to the partition $P = \{t_i\},\ 0 \leq i \leq n$, of $[a, b]$.

   (b) A **norm** $\| \cdot \|$ on a vector space $\mathcal{V}$.

   (c) The **interior** $int(A)$ of a subset A of the metric space (M, d).

   (d) A subset F of the metric space (M, d) is **closed**.

   (e) An **accumulation point** x of the subset A of the metric space (M, d).

2. Give an example of each of FIVE of the following SIX: (20 pts.)

   (a) A family of open sets whose intersection is not open.

   (b) A function with an infinite number of points of discontinuity that is integrable on $[0, 1]$.

   (c) A function with an infinite number of points of discontinuity that is NOT integrable on $[0, 1]$.

   (d) A bounded subset S of a metric space, such that S is not totally bounded.

   (e) A metric space that is not complete.

   (f) Two sets A and B such that $cl(A \cap B) \neq cl(A) \cap cl(B)$.

Exam continues. Please TURN OVER.

3. For each of the following sets S, find its **1. interior** $int(S)$, **2. boundary** $bd(S)$, **3. closure** $cl(S)$, **4. set of accumulation points** $acc(S)$, and **5. set of limit points**, and determine whether it is **6. open, closed, both or neither**: (30 pts.)

   (a) $\{x\} \subset M$ where M is a space with the discrete metric

   (b) $\{(0, y) \in \mathcal{R}^2 \mid 0 \leq y < 1\} \cup \{(x, y) \in \mathcal{R}^2 \mid x^2 + (y-2)^2 \leq 1\}$

   (c) the rationals: $\mathcal{Q} \subset \mathcal{R}$

   (d) the integers: $\mathcal{Z} \subset \mathcal{R}$

   (e) the space of integers $\mathcal{I} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ alone, with the standard metric $d(i, j) = |i - j|,\ i, j \in \mathcal{I}$, **NOT** as a subset of the reals

4. **Prove** ANY TWO OF the following THREE theorems: (30 pts.)

Theorem 1 *Every convergent sequence x_n in a metric space (M, d) is Cauchy.*

Theorem 2 *Every continuous function f on $[a, b]$ is integrable.*

Theorem 3 *If A is dense in $\mathcal{R}^n$, then every point of A is an accumulation point of both A and $\mathcal{R}^n$.*

(Recall: a set A is dense in $\mathcal{R}^n$ if and only if $cl(A) = \mathcal{R}^n$.)