Math 323 — **Quiz 1** — October 16, 2013

Please PRINT your name below. Write clearly and cross-out work not to be graded. The questions are to be answered directly on this paper as indicated. Total: 50 pts. plus extra credit.

Name:____

1. Give **definitions** of each of the following: (15 pts.)

   (a) The sequence of reals (s_n) **converges** to $s \in R$.

   (b) $\liminf s_n$ of a sequence (s_n) of reals.

   (c) A series $\sum_{n=1}^{\infty} a_n$ of real numbers a_n satisfies the **Cauchy criterion**. Do NOT presume the definition of partial sum.

2. **True or false?** Indicate your answer for each statement by placing **T** or **F** in the blank at the start. **Extra credit:** give a counterexample at the end of each false statement. (10 pts.)

_______ Every bounded sequence of reals converges.

_______ Every monotone sequence of reals has a convergent subsequence.

_______ Every convergent sequence of reals is bounded.

_______ $\limsup s_n$ exists for any sequence (s_n) of reals.

_______ If $\lim s_n = 0$, then $\lim(1/s_n) = +\infty$.

Quiz continues on reverse. Please TURN OVER.

3. **Define** what it means for a sequence (s_n) of reals to have limit (i.e. diverge to) $+\infty$, and then **use the negation of the definition to prove** that $\lim s_n \neq +\infty$ when $s_n = \frac{n}{n+1}$: (15 pts.)

4. **Define** what it means for a sequence (s_n) of reals to be Cauchy, and then **use the definition to prove** that $s_n = \frac{1}{n}$ is Cauchy. (10 pts.)

Extra credit: what theorems might you alternatively use to prove this sequence is Cauchy?