Math 323 — **QUIZ 1** — March 16, 2005

Please PRINT your name and ID# on the cover of the exam booklet. Write clearly and cross-out work not to be graded. Total: 50 pts.

1. **Give definitions** of ANY 4 of the following five: (20 pts.)

   (a) A sequence (s_n) of reals is **Cauchy**.

   (b) $\limsup s_n$ of a sequence (s_n) of reals.

   (c) A nonempty set S of reals is **bounded above**.

   (d) The $\inf S = -\infty$, for a set S of reals.

   (e) A series $\sum_{n=1}^{\infty} a_n$ of real numbers a_n **converges**.

2. **State** and then **use** the definition to prove that $\lim s_n = 0$ when (15 pts.)

$$s_n = \frac{4n}{7n^3 + 5}$$

3. Give an **example** of each of the following, or state that it is **impossible** to do so: (5 pts.)

   (a) an **unbounded sequence** with a convergent subsequence

   (b) a **bounded sequence** with NO convergent subsequence

   (c) a **sequence of irrationals** converging to a rational

   (d) a **sequence** (s_n) such that $\limsup s_n \leq \liminf s_n$

   (e) a **sequence of positive numbers** whose limit is negative

4. **True or false**: (5 pts.)

   (a) If $a_n < b_n$, $\forall n$, and $\sum b_n$ converges, then so does $\sum a_n$.

   (b) If $s_n \to 0$ is a sequence of positive reals, then $\lim(1/s_n) = \infty$.

   (c) For any nonempty set S of reals, $\inf(S) = \sup(-S)$.

   (d) Convergent sequences are bounded.

   (e) Monotone sequences converge.