4.3

2) a) f is Increasing : $(0, 2.6)$, $(4, 5.5)$

b) f is decreasing : $(-1, 0)$, $(2.6, 4)$, $(5.5, 7)$, $(7, 8)$

c) f is Concave Up $\{C.U.\}$: $(-1, 2)$, $(7, 8)$

d) f is Concave Down $\{C.D.\}$: $(2, 4)$, $(4, 7)$

e) point of inflection : at $x = 2$

---

6) $r(x) = 2.6x^2 - 7.28x + 4.9$

$$\frac{dr}{dx} = 2.6[2x] - 7.28[1] + [0] = 5.4x - 7.28$$

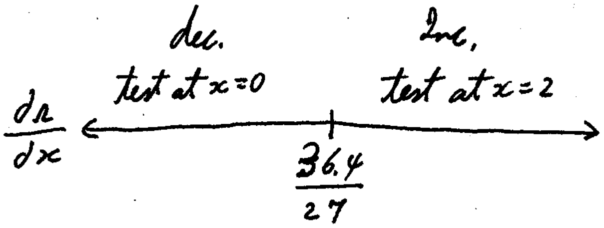
C.P.:

$$0 = \frac{dr}{dx} = 5.4x - 7.28$$

$$0 = 5.4x - 7.28$$

$$7.28 = 5.4x$$

$$x = \frac{7.28}{5.4} = \frac{72.8}{54} = \frac{36.4}{27} \approx 1.35$$



local min

test at $x=0$

$$\left.\frac{dr}{dx}\right|_{x=0} = 5.4(0) - 7.28 = -7.28 < 0 \text{ dec.}$$

test at $x=2$

$$\left.\frac{dr}{dx}\right|_{x=2} = 5.4(2) - 7.28 = 10.8 - 7.28 = 3.52 > 0 \text{ Inc.}$$

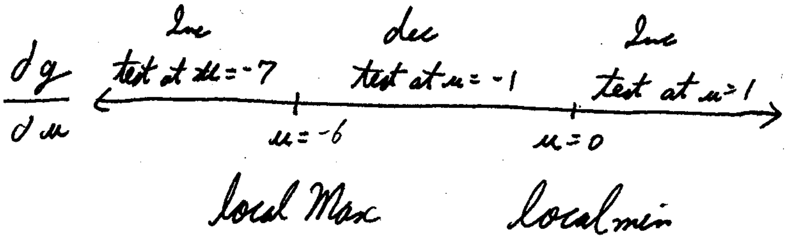
local min at $x = \frac{36.4}{27}$

8) $g(u) = 0.2u^3 + 1.8u^2 + 141$

$$\frac{dg}{du} = 0.2[3u^2] + 1.8[2u] = 0.6u^2 + 3.6u$$

C.P.:

$$0 = \frac{dg}{du} = 0.6u^2 + 3.6u$$



$$0 = 0.6u(u+6)$$

$0.6u = 0$ | $u + 6 = 0$

$u = 0$ | $u = -6$

test at $u = -7$

$$\left.\frac{dg}{du}\right|_{u=-7} = 0.6(-7)^2 + 3.6(-7) = 0.6(49) - 25.2 = 29.4 - 25.2 = 4.2 > 0 \text{ Inc.}$$

test at $u = -1$

$$\left.\frac{dg}{du}\right|_{u=-1} = 0.6(-1)^2 + 3.6(-1) = 0.6(1) - 3.6 = -3.0 < 0 \text{ dec.}$$

test at $x = 1$

$$\left.\frac{dg}{du}\right|_{u=1} = 0.6(1)^2 + 3.6(1) = 0.6 + 3.6 = 4.2 > 0 \text{ Inc.}$$

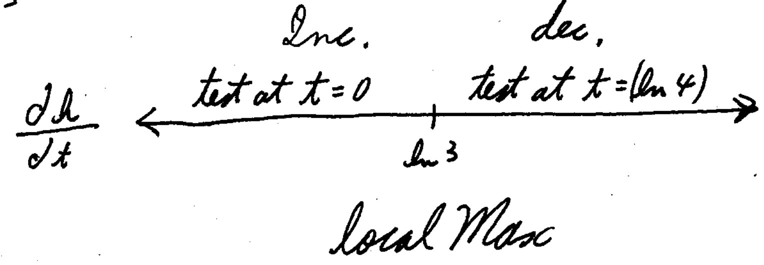
local Max at $u = -6$ local min at $u = 0$

---

10) $h(t) = 3t - e^t + 5$

$$\frac{dh}{dt} = 3[1] - [e^t(1)] + [0] = 3 - e^t$$

C.P.:



$$0 = \frac{dh}{dt} = 3 - e^t$$

$$0 = 3 - e^t$$

$$e^t = 3$$

$$\Downarrow$$

$$t = \ln 3 \approx 1.098$$

10) continued...

test at $t=0$

$$\left.\frac{dh}{dt}\right|_{t=0} = 3-e^{(0)} = 3-1 = 2 > 0 \text{ Inc}$$

test at $t=(\ln 4)$

local Max at $t=\ln 3$

$$\left.\frac{dh}{dt}\right|_{t=\ln 4} = 3-e^{(\ln 4)} = 3-4 = -1 < 0 \text{ dec}$$

---

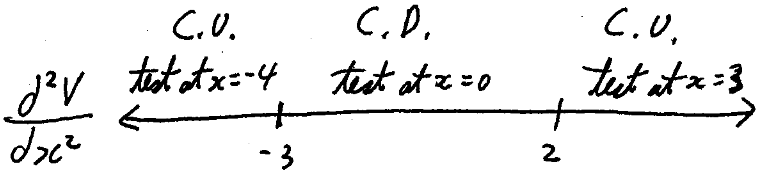
12) $V(x) = x^4 + 2x^3 - 36x^2 + 6$

$$\frac{dV}{dx} = [4x^3] + 2[3x^2] - 36[2x] + [0] = 4x^3 + 6x^2 - 72x$$

$$\frac{d^2V}{dx^2} = 4[3x^2] + 6[2x] - 72[1] = 12x^2 + 12x - 72$$

I.P. :

$$0 = \frac{d^2V}{dx^2} = 12x^2 + 12x - 72$$



$$0 = 12(x^2 + x - 6)$$

$$0 = 12(x+3)(x-2)$$

Concave Up {C.U.}: $(-\infty, -3) \cup (2, \infty)$

Concave Down {C.D.}: $(-3, 2)$

Inflection points ; $x=-3$, $x=2$

$x+3=0 \quad | \quad x-2=0$

$x=-3 \quad | \quad x=2$

test at $x=-4$

$$\left.\frac{d^2V}{dx^2}\right|_{x=-4} = 12(-4)^2 + 12(-4) - 72 = 12(16) - 48 - 72 = 192 - 120 = 72 > 0 \text{ C.U.}$$

test at $x=0$

$$\left.\frac{d^2V}{dx^2}\right|_{x=0} = 12(0)^2 + 12(0) - 72 = -72 < 0 \text{ C.D.}$$

test at $x=3$

$$\left.\frac{d^2V}{dx^2}\right|_{x=3} = 12(3)^2 + 12(3) - 72 = 12(9) + 36 - 72 = 108 - 36 = 72 > 0 \text{ C.U.}$$

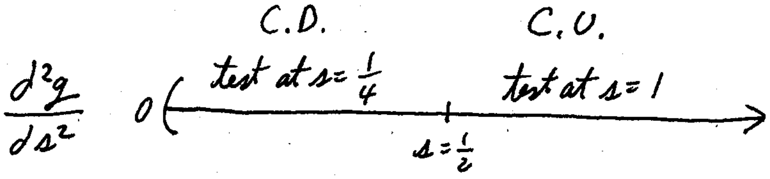
14) $g(s) = s^2 - s \ln s$ $\{\text{domain}: (0,\infty)\}$

$$\frac{dg}{ds} = [2s] - \{[1](\ln s) + (s)[\tfrac{1}{s}(1)]\} = 2s - \{\ln s + 1\} = 2s - \ln s - 1$$

$$\frac{d^2g}{ds^2} = 2[1] - [\tfrac{1}{s}(1)] - [0] = 2 - \frac{1}{s} = \frac{2s-1}{s}$$

I.P.:

$$0 = \frac{d^2g}{ds^2} = \frac{2s-1}{s}$$



$0 = 2s - 1$

$1 = 2s$

$\frac{1}{2} = s$

Inflection point: $s = \frac{1}{2}$

Concave Up {C.U.}: $(\frac{1}{2}, \infty)$

Concave Down {C.D.}: $(0, \frac{1}{2})$

test at $s = \frac{1}{4}$

$$\left.\frac{d^2g}{ds^2}\right|_{s=\frac{1}{4}} = 2 - \frac{1}{(\frac{1}{4})} = 2 - 4 = -2 < 0 \text{ C.D.}$$

test at $s = 1$

$$\left.\frac{d^2g}{ds^2}\right|_{s=1} = 2 - \frac{1}{(1)} = 2 - 1 = 1 > 0 \text{ C.U.}$$

---

16) $f(x) = 4x^3 - 3x^2 - 18x + 4$ domain: $(-\infty, \infty)$

$$\frac{df}{dx} = 4[3x^2] - 3[2x] - 18[1] + [0] = 12x^2 - 6x - 18$$

$$\frac{d^2f}{dx^2} = 12[2x] - 6[1] - [0] = 24x - 6$$

C.P.:

$0 = \frac{df}{dx} = 12x^2 - 6x - 18$

$0 = 6(2x^2 - x - 3)$

$0 = 6(x+1)(2x-3)$

$x + 1 = 0$

$x = -1$

$2x - 3 = 0$

$\vdots$

$x = \frac{3}{2}$

I.P.:

$0 = \frac{d^2f}{dx^2} = 24x - 6$

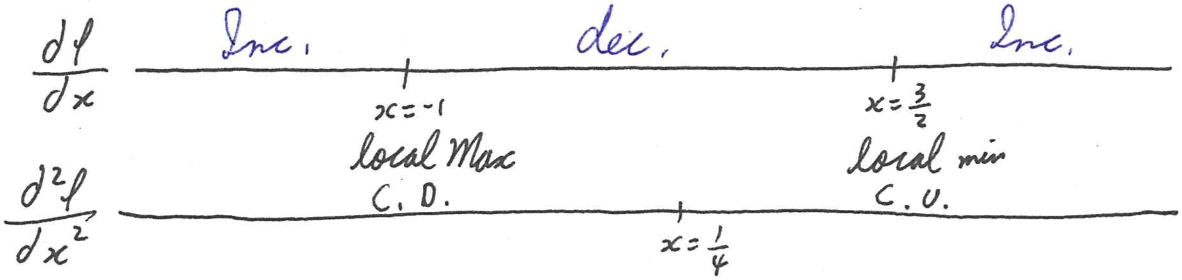
$0 = 24x - 6$

$6 = 24x$

$\frac{6}{24} = x$

$x = \frac{1}{4}$

16) continued...



it is more efficient if we test C.P. values in 2nd derivative. We can do this as long as C.P. and I.P. does not have the same values.

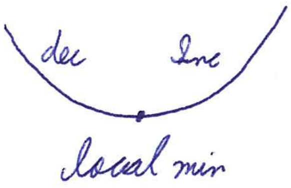
test C.P. $x=-1$

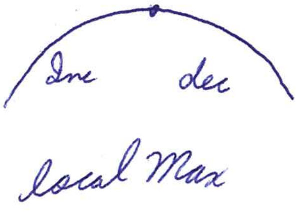
$$\left.\frac{d^2f}{dx^2}\right|_{x=-1} = 24(-1)-6 = -30 < 0 \text{ C.D. local Max}$$

test at C.P. $x=\frac{3}{2}$

$$\left.\frac{d^2f}{dx^2}\right|_{x=\frac{3}{2}} = 24\left(\frac{3}{2}\right)-6 = 12(3)-6 = 36-6 = 30 > 0 \text{ C.U. local min}$$

we can determine if the intervals is decreasing or Increasing by how local min and local Max is created.





a) Increasing: $(-\infty,-1) \cup (\frac{3}{2},\infty)$

decreasing: $(-1,\frac{3}{2})$

b) local Max: at $x=-1$, $f(-1)=4(-1)^3-3(-1)^2-18(-1)+4=-4-3+18+4=15$

$(-1,15)$

local min: at $x=\frac{3}{2}$, $f(\frac{3}{2})=4(\frac{3}{2})^3-3(\frac{3}{2})^2-18(\frac{3}{2})+4 = \frac{27}{2}-\frac{27}{4}-27+4$

$= \frac{54}{4}-\frac{27}{4}-\frac{108}{4}+\frac{16}{4} = \frac{-65}{4}$

$(\frac{3}{2},\frac{-65}{4})$

16) continued (2nd page)...

c) Concave Up {C.U.}: $(\frac{1}{4}, \infty)$

Concave Down {C.D.}: $(-\infty, \frac{1}{4})$

Inflection point: at $x = \frac{1}{4}$

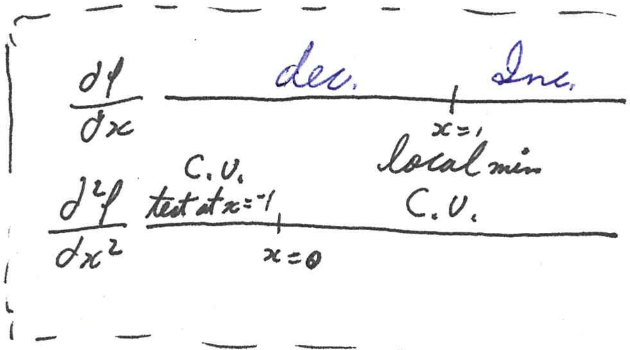
$f(\frac{1}{4}) = 4(\frac{1}{4})^3 - 3(\frac{1}{4})^2 - 18(\frac{1}{4}) + 4$

$= \frac{1}{16} - \frac{3}{16} - \frac{18}{4} + 4 = \frac{1}{16} - \frac{3}{16} - \frac{72}{16} + \frac{64}{16} = \frac{-10}{16} = \frac{-5}{8}$ $(\frac{1}{4}, \frac{-5}{8})$

18) $f(x) = x^4 - 4x + 1$ domain: $(-\infty, \infty)$

$\frac{df}{dx} = [4x^3] - 4[1] + [0] = 4x^3 - 4$

$\frac{d^2f}{dx^2} = 4[3x^2] - [0] = 12x^2$



C.P.:

$0 = \frac{df}{dx} = 4x^3 - 4$

$0 = 4(x^3 - 1)$

$0 = 4(x-1)(x^2 + x(1) + (1)^2)$

$x - 1 = 0$ not factorable in real number

$x = 1$

I.P.:

$0 = \frac{d^2f}{dx^2} = 12x^2$

$0 = 12x^2$

$0 = x^2$

$0 = x$

test at C.P. $x = 1$

$\left.\frac{d^2f}{dx^2}\right|_{x=1} = 12(1)^2 = 12 > 0$ C.U. local min

we need to test $x = -1$ to see if $x = 0$ is an inflection point.

$\left.\frac{d^2f}{dx^2}\right|_{x=-1} = 12(-1)^2 = 12 > 0$ C.U.

a) Increasing: $(1, \infty)$

decreasing: $(-\infty, 1)$

18) continued...

b) local min: at $x=1$ $f(1)=(1)^4-4(1)+1=-2$ $(1,-2)$

no local Max

c) Concave Up {C.U.}: $(-\infty,0)\cup(0,\infty)$

Concave Down {C.D.}: none

Inflection point: none because concavity does not change around $x=0$

---

20) $f(x)=\dfrac{x^2}{x^2+3}$ domain: $(-\infty,\infty)$ because $x^2+3=0$ does not have real number solution

$$\frac{df}{dx}=\frac{[2x](x^2+3)-(x^2)[2x]}{(x^2+3)^2}=\frac{2x^3+6x-2x^3}{(x^2+3)^2}=\frac{6x}{(x^2+3)^2}$$

$$\frac{d^2f}{dx^2}=\frac{\{[6]((x^2+3)^2)-(6x)[2(x^2+3)^1(2x)]\}}{((x^2+3)^2)^2}=\frac{6(x^2+3)\{[1](x^2+3)-(x)[2(2x)]\}}{(x^2+3)^4}$$

$$=\frac{6\{x^2+3-4x^2\}}{(x^2+3)^3}=\frac{6\{3-3x^2\}}{(x^2+3)^3}$$

C.P.:

$$0=\frac{df}{dx}=\frac{6x}{(x^2+3)^2}$$

$0=6x$

$0=x$

I.P.:

$$0=\frac{d^2f}{dx^2}=\frac{6\{3-3x^2\}}{(x^2+3)^3}$$

$0=6\{3-3x^2\}$

$0=18(1-x^2)$

$0=18(1+x)(1-x)$

$1+x=0$

$x=-1$

$1-x=0$

$x=1$

$\frac{df}{dx}$ dec. | Inc.

$x=0$

local min

$\frac{d^2f}{dx^2}$ C.D. test at $x=-2$ | C.U. | C.D. test at $x=2$

$x=-1$ $x=1$

20) continued...

test at C.P. $x = 0$

$$\left.\frac{d^2f}{dx^2}\right|_{x=0} = \frac{6\{3-3(0)^2\}}{((0)^2+3)^3} > 0 \quad \text{C.U. local min}$$

test at $x = -2$

$$\left.\frac{d^2f}{dx^2}\right|_{x=-2} = \frac{6\{3-3(-2)^2\}}{((-2)^2+3)^3} = \frac{6\{3-12\}}{(4+3)^3} < 0 \quad \text{C.D.}$$

test at $x = 2$

$$\left.\frac{d^2f}{dx^2}\right|_{x=2} = \frac{6\{3-3(2)^2\}}{((2)^2+3)^3} = \frac{6\{3-12\}}{(4+3)^3} < 0 \quad \text{C.D.}$$

a) Increasing: $(0, \infty)$

decreasing: $(-\infty, 0)$

b) local min: at $x = 0$ $\quad f(0) = \frac{(0)^2}{(0)^2+3} = \frac{0}{3} = 0 \quad (0,0)$

no local Max

c) Concave Up {C.U.}: $(-1, 1)$

Concave Down {C.D.}: $(-\infty, -1) \cup (1, \infty)$

Inflection points:

$f(-1) = \frac{(-1)^2}{(-1)^2+3} = \frac{1}{4} \quad (-1, 4)$ and $f(1) = \frac{(1)^2}{(1)^2+3} = \frac{1}{4} \quad (1, \frac{1}{4})$

---

22) $f(x) = x^2 e^x$ $\qquad$ domain: $(-\infty, \infty)$

$$\frac{df}{dx} = [2x](e^x) + (x^2)[e^x(1)] = 2xe^x + x^2e^x = (2x+x^2)e^x$$

22) continued...

$$\frac{d^2f}{dx^2} = [2+2x](e^x) + (2x+x^2)[e^x(1)]$$

$$= e^x\{[2+2x](1) + (2x+x^2)[1]\} = e^x\{2+2x+2x+x^2\}$$

$$= e^x\{x^2+4x+2\}$$

C.P.:

$$0 = \frac{df}{dx} = (2x+x^2)e^x$$

$$0 = x(2+x)e^x$$

$x = 0$ | $2+x=0$, $x=-2$ | $e^x = 0$ discard

I.P.:

$$0 = \frac{d^2f}{dx^2} = e^x\{x^2+4x+2\}$$

$$0 = e^x\{x^2+4x+2\}$$

$e^x = 0$ discard | $x^2+4x+2=0$

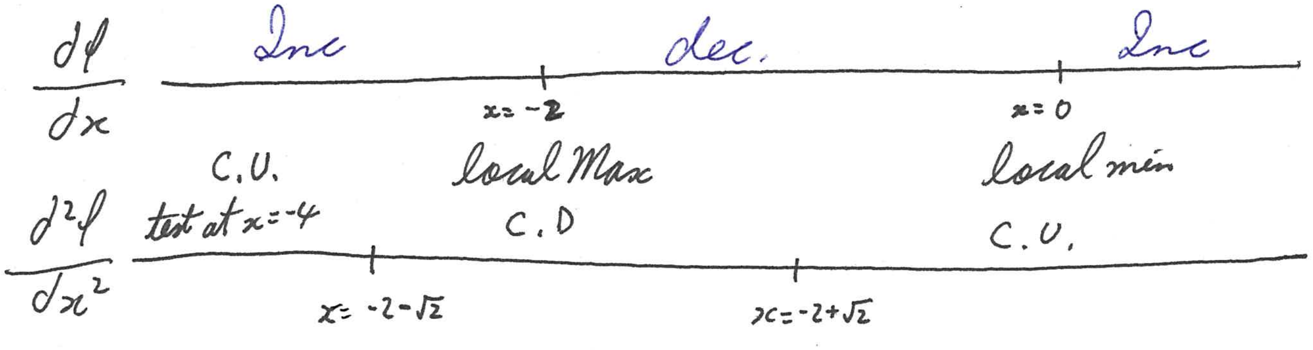
$$x = \frac{-(4) \pm \sqrt{(4)^2-4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{4(4-2)}}{2} = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$x = -2 \pm \sqrt{2}$$

$x = -2-\sqrt{2}$ less than -3

$x = -2+\sqrt{2}$ between -1 and 0



test at C.P. $x=-2$

$$\left.\frac{d^2f}{dx^2}\right|_{x=-2} = e^{(-2)}\{(-2)^2+4(-2)+2\} = \frac{1}{e^2}\{4-8+2\} = \frac{-2}{e^2} < 0 \text{ C.D local Max}$$

22) continued (2nd page)...

test at $x = 0$

$$\frac{d^2f}{dx^2}\bigg|_{x=0} = e^{(0)}\{(0)^2+4(0)+2\} = 1\{2\} = 2 > 0 \text{ C.U. local min}$$

test at $x = -4$

$$\frac{d^2f}{dx^2}\bigg|_{x=-4} = e^{(-4)}\{(-4)^2+4(-4)+2\} = \frac{1}{e^4}\{16-16+2\} = \frac{2}{e^4} > 0 \text{ C.U.}$$

a) Increasing: $(-\infty, -2) \cup (0, \infty)$

decreasing: $(-2, 0)$

b) local Max: at $x=-2$ $f(-2) = (-2)^2 e^{(-2)} = \frac{4}{e^2}$ $\left(-2, \frac{4}{e^2}\right)$

local min: at $x=0$ $f(0) = (0)^2 e^{(0)} = (0)(1) = 0$ $(0, 0)$

c) Concave Up {C.U.}: $(-\infty, -2-\sqrt{2}) \cup (-2+\sqrt{2}, \infty)$

Concave Down {C.D.}: $(-2-\sqrt{2}, -2+\sqrt{2})$

Inflection points:

$$f(-2-\sqrt{2}) = (-2-\sqrt{2})^2 e^{(-2-\sqrt{2})} = (4+4\sqrt{2}+(2))e^{-(2+\sqrt{2})} = \frac{6+4\sqrt{2}}{e^{(2+\sqrt{2})}}$$

$$\left(-2-\sqrt{2}, \frac{6+4\sqrt{2}}{e^{(2+\sqrt{2})}}\right)$$

$$f(-2+\sqrt{2}) = (-2+\sqrt{2})^2 e^{(-2+\sqrt{2})} = (4-4\sqrt{2}+(2))e^{-(2-\sqrt{2})} = \frac{6-4\sqrt{2}}{e^{(2-\sqrt{2})}}$$

$$\left(-2+\sqrt{2}, \frac{6-4\sqrt{2}}{e^{(2-\sqrt{2})}}\right)$$

26) $g(x)=200+8x^3+x^4$ domain: $(-\infty,\infty)$

$$\frac{dg}{dx}=[0]+8[3x^2]+[4x^3]=4x^3+24x^2$$

$$\frac{d^2g}{dx^2}=4[3x^2]+24[2x]=12x^2+48x$$

C.P.:

$0=\frac{dg}{dx}=4x^3+24x^2$

$0=4x^2(x+6)$

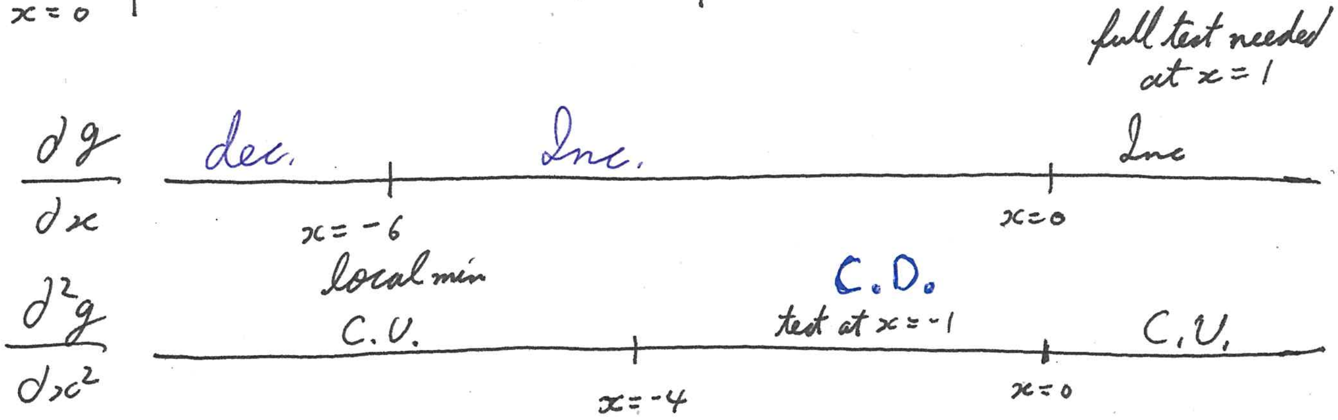
$4x^2=0$ | $x+6=0$
$x^2=0$ | $x=-6$
$x=0$

I.P.:

$0=\frac{d^2g}{dx^2}=12x^2+48x$

$0=12x(x+4)$

$12x=0$ | $x+4=0$
$x=0$ | $x=-4$



test at C.P. $x=-6$

$$\left.\frac{d^2g}{dx^2}\right|_{x=-6}=12(-6)^2+48(-6)=432-288=144>0 \text{ C.U. local min}$$

full test at $x=1$

$$\left.\frac{dg}{dx}\right|_{x=1}=4(1)^3+24(1)^2=28>0 \text{ Inc}$$

$$\left.\frac{d^2g}{dx^2}\right|_{x=1}=12(1)^2+48(1)=60>0 \text{ C.U.}$$

this is an I.P. with slope 0

test at $x=-1$ "not needed"

$$\left.\frac{d^2g}{dx^2}\right|_{x=-1}=12(-1)^2+48(-1)=-36<0$$

C.D.

can determine by deduction

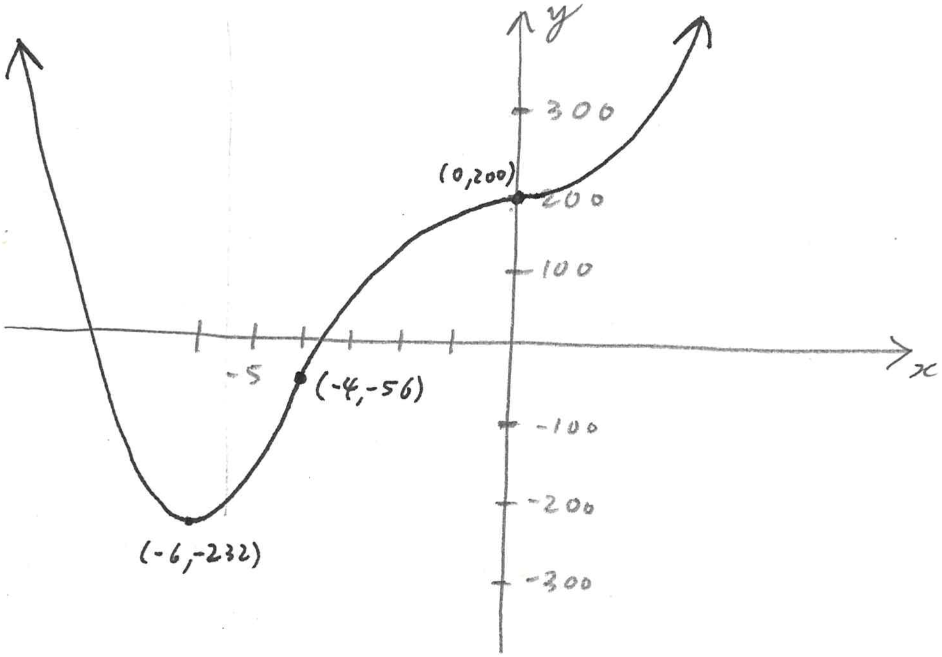
26) continued...

we need coordinates for $x = -6$, $x = -4$ and $x = 0$

$g(-6) = 200 + 8(-6)^3 + (-6)^4 = 200 - 1728 + 1296 = -232$ $(-6, -232)$

$g(-4) = 200 + 8(-4)^3 + (-4)^4 = 200 - 512 + 256 = -56$ $(-4, -56)$

$g(0) = 200 + 8(0)^3 + (0)^4 = 200$ $(0, 200)$



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28) $h(x) = (x^2 - 1)^3$ domain: $(-\infty, \infty)$

$$\frac{dh}{dx} = \left[3(x^2-1)^2(2x)\right] = 6x(x^2-1)^2$$

$$\frac{d^2h}{dx^2} = \left\{[6]\left((x^2-1)^2\right) + (6x)\left[2(x^2-1)^1(2x)\right]\right\}$$

$$= 6(x^2-1)\left\{[1](x^2-1) + (x)\left[2(2x)\right]\right\}$$

$$= 6(x^2-1)\left\{x^2 - 1 + 4x^2\right\}$$

$$= 6(x^2-1)\left\{5x^2 - 1\right\}$$

28) continued...

C.P.:

$$0 = \frac{dh}{dx} = 6x\,(x^2-1)^2$$

$$0 = 6x\,(x^2-1)^2$$

$6x = 0$ | $(x^2-1)^2 = 0$

$x = 0$ | $x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x+1=0$ | $x-1=0$

$x=-1$ | $x=1$

I.P.:

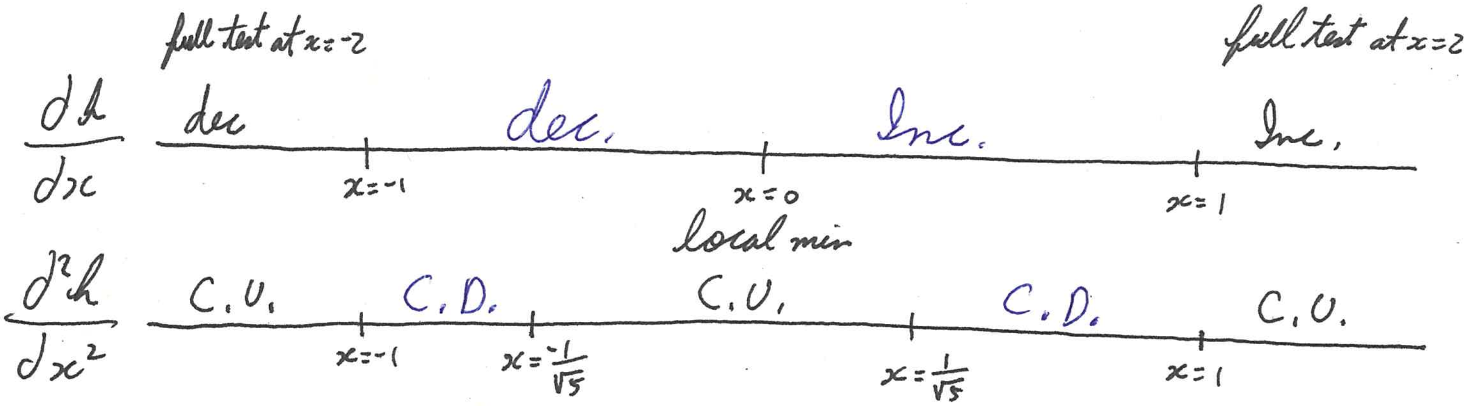
$$0 = \frac{d^2h}{dx^2} = 6(x^2-1)\,\{5x^2-1\}$$

$$0 = 6(x^2-1)\,\{5x^2-1\}$$

$$0 = 6(x+1)(x-1)(\sqrt{5}x+1)(\sqrt{5}x-1)$$

$x+1=0$ | $x-1=0$ | $\sqrt{5}x+1=0$ | $\sqrt{5}x-1=0$

$x=-1$ | $x=1$ | $x=\frac{-1}{\sqrt{5}}$ | $x=\frac{1}{\sqrt{5}}$



test at C.P. $x=0$

$$\left.\frac{d^2h}{dx^2}\right|_{x=0} = 6((0)^2-1)\{5(0)^2-1\} = 6(-1)\{-1\} = 6 > 0 \text{ C.U. local min}$$

full test at $x=-2$

$$\left.\frac{dh}{dx}\right|_{x=-2} = 6(-2)((-2)^2-1)^2 = 6(-2)(3)^2 < 0 \text{ decreasing}$$

this is IP with slope 0

$$\left.\frac{d^2h}{dx^2}\right|_{x=-2} = 6((-2)^2-1)\{5(-2)^2-1\} = 6(3)\{19\} > 0 \text{ C.U.}$$

full test at $x=2$

$$\left.\frac{dh}{dx}\right|_{x=2} = 6(2)((2)^2-1)^2 = 6(2)(3)^2 > 0 \text{ Increasing}$$

this is IP with slope 0

$$\left.\frac{d^2h}{dx^2}\right|_{x=2} = 6((2)^2-1)\{5(2)^2-1\} = 6(3)\{19\} > 0 \text{ C.U.}$$

28) continued (2nd page)...

we need coordinates for $x=-1$, $x=\frac{-1}{\sqrt{5}}$, $x=0$, $x=\frac{1}{\sqrt{5}}$, $x=1$

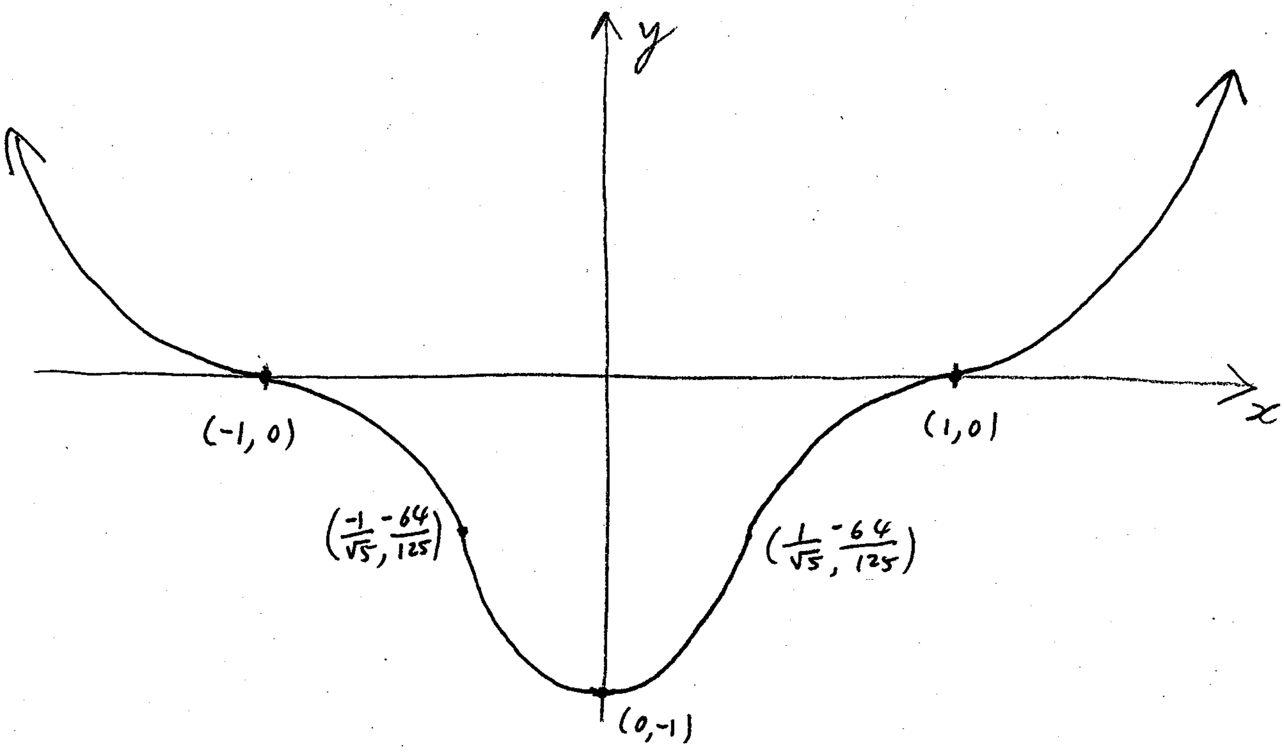
$h(-1)=((-1)^2-1)^3=(1-1)^3=(0)^3=0 \quad (-1,0)$

$h(\frac{-1}{\sqrt{5}})=((\frac{-1}{\sqrt{5}})^2-1)^3=(\frac{1}{5}-1)^3=(\frac{-4}{5})^3=\frac{-64}{125} \quad (\frac{-1}{\sqrt{5}},\frac{-64}{125})$

$h(0)=((0)^2-1)^3=(-1)^3=-1 \quad (0,-1)$

$h(\frac{1}{\sqrt{5}})=((\frac{1}{\sqrt{5}})^2-1)^3=(\frac{1}{5}-1)^3=(\frac{-4}{5})^3=\frac{-64}{125} \quad (\frac{1}{\sqrt{5}},\frac{-64}{125})$

$h(1)=((1)^2-1)^3=(1-1)^3=(0)^3=0 \quad (1,0)$



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32) a) f is Increasing on $(2,4) \cup (6,\infty)$ because the graph of $f'(x)$ is above x-axis which makes the value of $f'(x)=\frac{df}{dx}>0$.

32) continued...

b) local Maximum at $x=4$ because $\overset{x<4}{\underset{\text{Inc}}{\frac{df}{dx}>0}}$ $\quad$ $\overset{4<x}{\underset{\text{dec}}{\frac{df}{dx}<0}}$

local minimum at $x=2$ because $\overset{x<2}{\underset{\text{dec}}{\frac{df}{dx}<0}}$ $\quad$ $\overset{2<x}{\underset{\text{Inc}}{\frac{df}{dx}>0}}$

at $x=6$ because $\overset{x<6}{\underset{\text{dec}}{\frac{df}{dx}<0}}$ $\quad$ $\overset{6<x}{\underset{\text{Inc}}{\frac{df}{dx}>0}}$

c) Concave Up {C.U.}: $(1,3)\cup(5,7)\cup(8,\infty)$
because the slope of the graph of $f'(x)=\frac{df}{dx}$ is positive and thus makes $\frac{d^2f}{dx^2}>0$ which is C.U.

Concave Down {C.D}: $(0,1)\cup(3,5)\cup(7,8)$
because the slope of the graph of $f'(x)=\frac{df}{dx}$ is negative and thus makes $\frac{d^2f}{dx^2}<0$ which is C.D.

d) the x-coordinates of the inflection points are
$x=1,\ x=3,\ x=5,\ x=7,\ x=8$
because these are the critical points of the graph of $f'(x)=\frac{df}{dx}$ (which is already 1st derivative) and we need 1 more derivative to get actual 2nd derivative. C.P. (for $f'(x)=\frac{df}{dx}$): $0=\frac{d}{dx}(f'(x))=\frac{d}{dx}\left(\frac{df}{dx}\right)=\frac{d^2f}{dx^2}$.