4.1 revised 10/16/2023

2) a) $A = \pi r^2$

$\left[1 \frac{dA}{dt}\right] = \pi \left[2r \frac{dr}{dt}\right]$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

b) $\frac{dr}{dt} = 1 \text{ m/sec}$; when $r = 30 \text{ cm}$, $\frac{dA}{dt} = ?$

$\frac{dA}{dt} = 2\pi (30\text{cm})(1\text{m/sec}) = 60\pi \text{ m}^2/\text{sec}$

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4) $\frac{dl}{dt} = 8 \text{ cm/sec}$, $\frac{dw}{dt} = 3 \text{ cm/sec}$; when $l = 20 \text{ cm}$ and $w = 10 \text{ cm}$, $\frac{dA}{dt} = ?$

$A = lw$

$\left[1 \frac{dA}{dt}\right] = \left[1 \frac{dl}{dt}\right](w) + (l)\left[1 \frac{dw}{dt}\right]$

$\frac{dA}{dt} = w \frac{dl}{dt} + l \frac{dw}{dt}$

$\frac{dA}{dt} = (10\text{cm})(8\text{cm/sec}) + (30\text{cm})(3\text{cm/sec})$

$= 80 \text{cm}^2/\text{sec} + 90 \text{cm}^2/\text{sec}$

$= 170 \text{cm}^2/\text{sec}$

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6) $x^2 + y^2 = 25$ $\quad \frac{dy}{dt} = 6$; when $y = 4$, $\frac{dx}{dt} = ?$

when $y = 4$:

$x^2 + (4)^2 = 25$

$x^2 + 16 = 25$

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = +\sqrt{9}$

$x = +3$

$x^2 + y^2 = 25$

$\left[2x \frac{dx}{dt}\right] + \left[2y \frac{dy}{dt}\right] = [0]$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-2y}{2x} \frac{dy}{dt}$

$= \frac{-y}{x} \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-(4)}{(3)}(6)$

$= -(4)(2)$

$\frac{dx}{dt} = -8$

8) $y = \sqrt{1+x^3}$ point $(2,3)$, $\frac{dy}{dt} = +4$ cm/sec, $\frac{dx}{dt} = ?$

$y = (1+x^3)^{\frac{1}{2}}$

$\left[1\frac{dy}{dt}\right] = \left[\frac{1}{2}(1+x^3)^{-\frac{1}{2}}\left(3x^2\frac{dx}{dt}\right)\right]$

$\frac{dy}{dt} = \frac{3x^2}{2\sqrt{1+x^3}}\frac{dx}{dt}$

$\frac{2\sqrt{1+x^3}}{3x^2}\frac{dy}{dt} = \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{2\sqrt{1+(2)^3}}{3(2)^2}(4\text{ cm/sec})$

$= \frac{2\sqrt{9}}{3(2)^2}(4\text{ cm/sec})$

$= \frac{2(3)}{3(2)^2}(4\text{ cm/sec})$

$\frac{dx}{dt} = 2$ cm/sec

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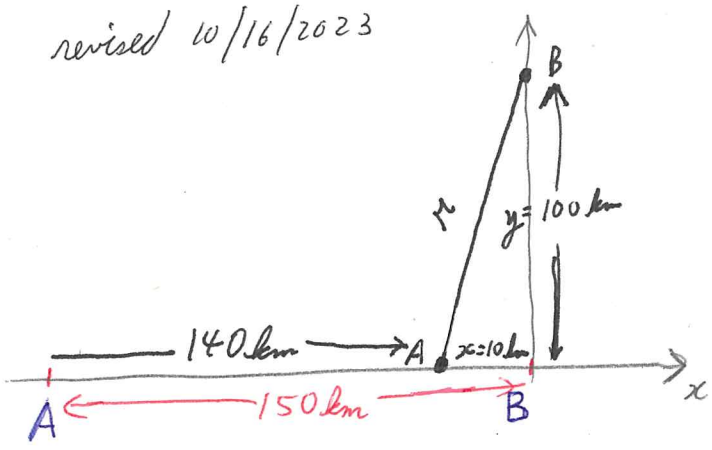
14) revised 10/16/2023



at noon

ship A moves along x-axis and $\frac{dA}{dt} = \frac{dx}{dt} = -35$ km/hr

ship B moves along y-axis and $\frac{dB}{dt} = \frac{dy}{dt} = +25$ km/sec

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at 4:00PM {boats travelled}

horizontal $= |(-35\text{ km/hr})|(4\text{hr}) = 140$ km

vertical $= |(+25\text{ km/hr})|(4\text{hr}) = 100$ km

$r^2 = x^2 + y^2$

$\left[2r\frac{dr}{dt}\right] = \left[2x\frac{dx}{dt}\right] + \left[2y\frac{dy}{dt}\right]$

$\frac{dr}{dt} = \frac{2x}{2r}\frac{dx}{dt} + \frac{2y}{2r}\frac{dy}{dt}$

$\frac{dr}{dt} = \frac{x}{r}\frac{dx}{dt} + \frac{y}{r}\frac{dy}{dt}$

$r^2 = (10)^2 + (100)^2$ "revised 10/16/2023"

$r^2 = 100 + 10000$

$r^2 = 10100$

$r = \pm\sqrt{10100} = \pm\sqrt{101(100)}$

$r = \pm 10\sqrt{101}$

$r = +10\sqrt{101}$ km {distance is always +}

14) continued...

$$\frac{dr}{dt} = \frac{(10\ km)}{(10\sqrt{101}\ km)}(-35\ km/hr) + \frac{(100\ km)}{(10\sqrt{101}\ km)}(+25\ km/hr)$$

$$= \frac{-35}{\sqrt{101}}\ km/hr + \frac{250}{\sqrt{101}}\ km/hr$$

$$\frac{dr}{dt} = \frac{215}{\sqrt{101}}\ km/hr$$

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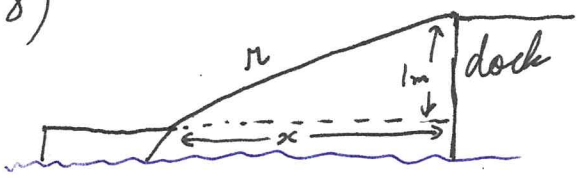
18)



$$r^2 = x^2 + (1)^2$$

$$\left[2r\frac{dr}{dt}\right] = \left[2x\frac{dx}{dt}\right] + [0]$$

$$\frac{2r}{2x}\frac{dr}{dt} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{r}{x}\frac{dr}{dt}$$

$\frac{dr}{dt} = (-1)\ m/sec$

when $x = 8\ m$, $\frac{dx}{dt} = ?$

$$r^2 = (8\ m)^2 + (1)^2$$

$$r^2 = 64 + 1 = 65$$

$$r = \pm\sqrt{65} \Rightarrow r = +\sqrt{65}$$

$$\frac{dx}{dt} = \frac{(+\sqrt{65}\ m)}{(8\ m)}(-1\ m/sec)$$

$$\frac{dx}{dt} = \frac{-\sqrt{65}}{8}\ m/sec$$

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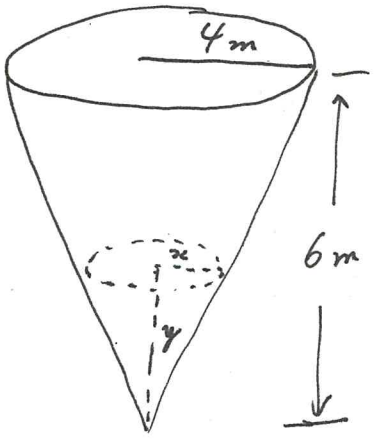
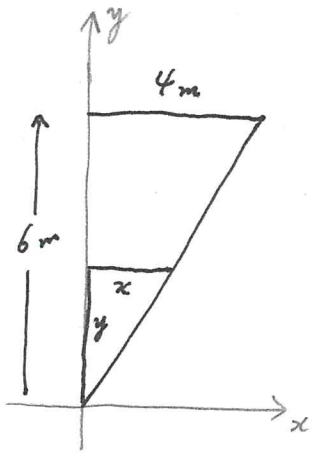
20)



side view



let y be height of this cone

x be the radius of the circle

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi x^2 y$$

20) continued...

water level is rising at rate of 20 cm/sec ⇒

$\frac{dy}{dt} = 20\,cm/sec = 0.02\,m/sec = \frac{2}{100}\,m/sec$

The rate of change of radius of the circle is not given. Therefore we need to use the side view in order to convert our Volume equation to have y (height) visible only.

Side view shows us that we have similar (proportional) triangles where we can set up proportion.

$$\frac{x}{4m} = \frac{y}{6m} \Rightarrow x = \frac{4m}{6m}y \Rightarrow x = \frac{2}{3}y\ m$$

$V = \frac{1}{3}\pi x^2 y$

$V = \frac{1}{3}\pi\left(\frac{2}{3}y\right)^2 y$

$V = \frac{1}{3}\pi\left(\frac{4}{9}y^2\right)y$

$V = \frac{4\pi}{27}y^3$

$\left[\frac{dV}{dt}\right] = \frac{4\pi}{27}\left[3y^2\frac{dy}{dt}\right]$

$\frac{dV}{dt} = \frac{4\pi}{9}y^2\frac{dy}{dt}$

when $y = 2m$, $\frac{dy}{dt} = \frac{2}{100}\,m/sec$

$\frac{dV}{dt} = ?$

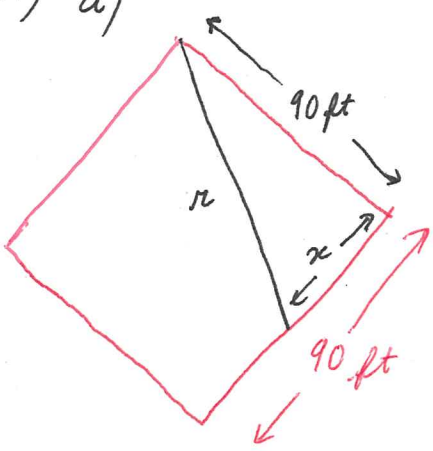
$\frac{dV}{dt} = \frac{4\pi}{9}(2m)^2\left(\frac{2}{100}\,m/sec\right)$

$= \frac{4\pi}{9}(4m^2)\left(\frac{1}{50}\,m/sec\right)$

$= \frac{4\pi}{9}(2m^2)\left(\frac{1}{25}\,m/sec\right)$

$\frac{dV}{dt} = \frac{8\pi}{225}\,m^3/sec$

24) a)



$\frac{dx}{dt} = (-24)$ ft/sec; $x = 45$ ft, $\frac{dr}{dt} = ?$

$r^2 = x^2 + (90)^2$

$\left[2r\frac{dr}{dt}\right] = \left[2x\frac{dx}{dt}\right] + [0]$

$\frac{dr}{dt} = \frac{2x}{2r}\frac{dx}{dt}$

$\frac{dr}{dt} = \frac{x}{r}\frac{dx}{dt}$

when $x = 45$ ft

$r^2 = (45)^2 + (90)^2$

$r^2 = (45)^2 + (2(45))^2$

$r^2 = (45)^2 + (2)^2(45)^2$

$r = +\sqrt{(45)^2 + (2)^2(45)^2}$

$r = +\sqrt{(45)^2(1 + (2)^2)}$

$r = \sqrt{(45)^2}\sqrt{(1+4)}$

$r = 45\sqrt{5}$ ft

$\frac{dr}{dt} = \frac{(45\text{ ft})}{(45\sqrt{5}\text{ ft})}(-24)$ ft/sec $= \frac{-24}{\sqrt{5}}$ ft/sec

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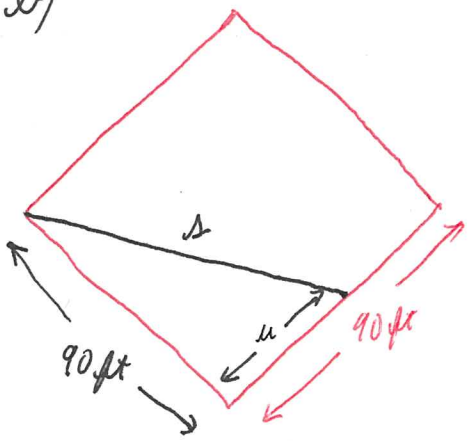
b)



$\frac{du}{dt} = (+24)$ ft/sec; $u = 45$ ft, $\frac{ds}{dt} = ?$

$s^2 = u^2 + (90)^2$

$\left[2s\frac{ds}{dt}\right] = \left[2u\frac{du}{dt}\right] + [0]$

$\frac{ds}{dt} = \frac{2u}{2s}\frac{du}{dt}$

$\frac{ds}{dt} = \frac{u}{s}\frac{du}{dt}$

when $u = 45$ ft

$s^2 = (45)^2 + (90)^2$

$s^2 = (45)^2 + (2(45))^2$

$s = +\sqrt{(45)^2 + (2(45))^2}$

$s = +\sqrt{(45)^2 + (2)^2(45)^2}$

$s = +\sqrt{(45)^2(1 + (2)^2)}$

$s = \sqrt{(45)^2}\sqrt{(1+4)}$

$s = 45\sqrt{5}$ ft

$\frac{ds}{dt} = \frac{(45\text{ ft})}{(45\sqrt{5}\text{ ft})}(+24)$ ft/sec $= \frac{+24}{\sqrt{5}}$ ft/sec