3.6

MATH 20900 only uses continuous (compounding interest) cases only for calculations. Therefore, we will only consider continuous case for this section topic.

$P(t)$: principal / population at time t

r: interest / growth / decay rate

t: time (pay attention of units of time used on each word problems)

$$P(t) = P_0 e^{rt}$$

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4) $P_0 = \$11.8$ million dollars (note: units is in million dollars)

$r = 14\% = 0.14$ $\quad$ t: in years

a) $\underline{\underline{P(t) = (11.8)e^{0.14t}}}$

b) $P(3.25) = ?$

$P(3.25) = \underline{\underline{(11.8)e^{0.14(3.25)} = 11.8e^{0.455} \text{ million dollars}}}$

c) $P(t) = \$20$ million, $t = ?$

$(20) = (11.8)e^{0.14t}$

$\frac{20}{11.8} = e^{0.14t}$

$\Downarrow$

$\ln\left(\frac{20}{11.8}\right) = 0.14t$

$\underline{\underline{t = \frac{\ln\left(\frac{20}{11.8}\right)}{0.14} \text{ years}}}$

6) $P_0 = 1650$, decrease 18% $\Rightarrow r = -18\% = -0.18$

t: in years

a) $P(t) = (1650)\, e^{-0.18t}$

b) $P(t) = 1000$, $t = ?$

$(1000) = (1650) e^{-0.18t}$

$\frac{1000}{1650} = e^{-0.18t}$

$\Downarrow$

$\ln\left(\frac{1000}{1650}\right) = -0.18t$

$t = \frac{\ln\left(\frac{1000}{1650}\right)}{-0.18}$ years

c) $\frac{dP}{dt} = 1650\left[e^{-0.18t}(-0.18)\right]$

$= -0.18(1650)\, e^{-0.18t}$

$\left.\frac{dP}{dt}\right|_{t=4} = ?$

$\left.\frac{dP}{dt}\right|_{t=4} = -0.18(1650)\, e^{-0.18(4)}$

$= (-297)\, e^{-0.72}$ eagle per year

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16) doubles every 20 minutes ($\frac{1}{3}$ hour) $\Rightarrow P(\frac{1}{3}) = 2P_0$

$P_0 = 60$ cells $\qquad t$: in hours

a) $P(\frac{1}{3}) = 2P_0 = 2(60)$, $r = ?$ $\qquad P(t) = P_0 e^{rt}$

$2(60) = (60)\, e^{r(\frac{1}{3})}$

$\frac{2(60)}{60} = e^{\frac{1}{3}r}$

$2 = e^{\frac{1}{3}r}$

$\Downarrow$

$\ln 2 = \frac{1}{3} r$

$3 \ln 2 = r$

$r = 3\ln 2 = \ln(2^3)$

b) $P(t) = (60)\, e^{(\ln(2^3))t}$

$P(t) = (60)\, e^{(3\ln(2))t}$

16) continued

c) $P(8) = ?$

$$P(8) = (60)e^{(3\ln(2))(8)} = (60)e^{(24\ln(2))} = (60)e^{\ln(2^{24})}$$

$$= \underline{\underline{(60)(2^{24}) \text{ cells}}}$$

d) $\left.\frac{dP}{dt}\right|_{t=8} = ?$

$$\frac{dP}{dt} = (60)\left[e^{(3\ln(2))t}(3\ln(2))\right]$$

$$= (180\ln(2))e^{(3\ln(2))t}$$

$$\left.\frac{dP}{dt}\right|_{t=8} = (180\ln(2))e^{(3\ln(2))(8)}$$

$$= (180\ln(2))e^{(24\ln(2))}$$

$$= (180\ln(2))e^{\ln(2^{24})}$$

$$= \underline{\underline{(180\ln(2))(2^{24}) \text{ cells per hour}}}$$

e) $P(t) = 20000,\ t = ?$

$$(20000) = (60)e^{(3\ln(2))t}$$

$$\frac{20000}{60} = e^{(3\ln(2))t}$$

$$\Downarrow$$

$$\ln\left(\frac{20000}{60}\right) = (3\ln(2))t$$

$$\frac{\ln\left(\frac{20000}{60}\right)}{3\ln(2)} = t$$

$$\underline{\underline{t = \frac{\ln\left(\frac{20000}{60}\right)}{3\ln(2)} \text{ hours}}}$$

20) $P(t) = P_0 e^{rt}$

after 2 hours, 600 bacteria $\Rightarrow P(2) = 600$

after 8 hours, 75000 bacteria $\Rightarrow P(8) = 75000$

20) continued part 2

$P(2)=600$

$\Downarrow$

$(600)=P_0 e^{r(2)}$

$600 = P_0 e^{2r}$

$\frac{600}{e^{2r}} = P_0$

$P(8)=75000$

$\Downarrow$

$(75000)=P_0 e^{r(8)}$

$75000 = P_0 e^{8r}$

$75000 = \left(\frac{600}{e^{2r}}\right)e^{8r}$

$\frac{75000}{600} = \frac{e^{8r}}{e^{2r}}$

$\frac{750}{6} = e^{(8r-2r)} = e^{6r}$

$\Downarrow$

$\ln\left(\frac{750}{6}\right) = 6r$

$\frac{\ln\left(\frac{750}{6}\right)}{6} = r$

$P_0 = \frac{600}{e^{2\left(\frac{\ln\left(\frac{750}{6}\right)}{6}\right)}}$

$= \frac{600}{e^{\left(\frac{\ln\left(\frac{750}{6}\right)}{3}\right)}}$

$= \frac{600}{e^{\frac{1}{3}\ln\left(\frac{750}{6}\right)}}$

$= \frac{600}{e^{\ln\left(\sqrt[3]{\frac{750}{6}}\right)}}$

$P_0 = \frac{600}{\sqrt[3]{\frac{750}{6}}}$ bacteria

b) $P(t) = \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)t}$

c) $P(5) = ?$

c) $P(5) = \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)(5)} = \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)e^{\left(\frac{5}{6}\ln\left(\frac{750}{6}\right)\right)}$

$= \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)e^{\ln\left(\left(\frac{750}{6}\right)^{\frac{5}{6}}\right)} = \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)\left(\left(\sqrt[6]{\frac{750}{6}}\right)^5\right)$ bacteria

20) continued part 3

d) $\left.\dfrac{dP}{dt}\right|_{t=5} = ?$

$$\frac{dP}{dt} = \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)\left[e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)t}\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)\right]$$

$$= \left(\frac{100}{\sqrt[3]{\frac{750}{6}}}\ln\left(\frac{750}{6}\right)\right)e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)t}$$

$$\left.\frac{dP}{dt}\right|_{t=5} = \left(\frac{100}{\sqrt[3]{\frac{750}{6}}}\ln\left(\frac{750}{6}\right)\right)e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)(5)} = \left(\frac{100}{\sqrt[3]{\frac{750}{6}}}\ln\left(\frac{750}{6}\right)\right)e^{\left(\frac{5}{6}\ln\left(\frac{750}{6}\right)\right)}$$

$$= \left(\frac{100}{\sqrt[3]{\frac{750}{6}}}\ln\left(\frac{750}{6}\right)\right)e^{\ln\left(\sqrt[6]{\frac{750}{6}}\right)^5} = \left(\frac{100}{\sqrt[3]{\frac{750}{6}}}\ln\left(\frac{750}{6}\right)\right)\left(\sqrt[6]{\frac{750}{6}}\right)^5 \text{ bacteria per hour}$$

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e) $P(t) = 200000,\ t = ?$

$$(200000) = \left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)t}$$

$$\frac{200000}{\left(\frac{600}{\sqrt[3]{\frac{750}{6}}}\right)} = e^{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)t}$$

$$\frac{2000\left(\sqrt[3]{\frac{750}{6}}\right)}{6} = e^{\left(\frac{1}{6}\ln\left(\frac{760}{6}\right)\right)t}$$

$$\Downarrow$$

$$\ln\left(\frac{2000\left(\sqrt[3]{\frac{750}{6}}\right)}{6}\right) = \left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)t$$

$$\frac{\ln\left(\frac{2000-\left(\sqrt[3]{\frac{750}{6}}\right)}{6}\right)}{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)} = t$$

$$t = \frac{\ln\left(\frac{2000-\left(\sqrt[3]{\frac{750}{6}}\right)}{6}\right)}{\left(\frac{1}{6}\ln\left(\frac{750}{6}\right)\right)} \text{ hours}$$

24) half-life of 5.0 days $\Rightarrow P(5) = \frac{1}{2}P_0$

$P(t) = P_0 e^{rt}$

a) $P_0 = 800\ mg$

$P(5) = \frac{1}{2}P_0 = \frac{1}{2}(800) = (800)e^{r(5)}$

$$\frac{\frac{1}{2}(800)}{(800)} = e^{5r}$$

$$\frac{1}{2} = e^{5r}$$

$$\Downarrow$$

$$\ln\left(\frac{1}{2}\right) = 5r$$

$$\frac{\ln\left(\frac{1}{2}\right)}{5} = r$$

$$P(t) = (800)e^{\left(\frac{\ln\left(\frac{1}{2}\right)}{5}\right)t}$$

b) $P(30) = ?$

$$P(30) = (800)e^{\left(\frac{\ln\left(\frac{1}{2}\right)}{5}\right)(30)}$$

$$= (800)e^{(6\ln\left(\frac{1}{2}\right))}$$

$$= (800)e^{\ln\left(\frac{1}{2}\right)^6}$$

$$= (800)\left(\frac{1}{2}\right)^6\ mg$$

c) $P(t) = 10\ mg$, $t = ?$

$$(10) = (800)e^{\left(\frac{\ln\left(\frac{1}{2}\right)}{5}\right)t}$$

$$\frac{10}{800} = e^{\left(\frac{\ln\left(\frac{1}{2}\right)}{5}\right)t}$$

$$\Downarrow$$

$$\ln\left(\frac{10}{800}\right) = \left(\frac{\ln\left(\frac{1}{2}\right)}{5}\right)t$$

$$\frac{5\ln\left(\frac{10}{800}\right)}{\ln\left(\frac{1}{2}\right)} = t$$

$$t = \frac{5\ln\left(\frac{10}{800}\right)}{\ln\left(\frac{1}{2}\right)}\ \text{hours}$$

d) omit sketch of the graph.