3.4 (revised 10/12/2023)

2) $y = \sqrt{4+3x} = (4+3x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(4+3x)^{-\frac{1}{2}}(3) = \frac{3}{2\sqrt{4+3x}}$$

---

4) $y = (x^3+x)^6$

$$\frac{dy}{dx} = 6(x^3+x)^5(3x^2+1) = 6(3x^2+1)(x^3+x)^5$$

---

6) $y = \sqrt{e^x} = (e^x)^{\frac{1}{2}} = e^{\frac{1}{2}x} = e^{\frac{x}{2}}$

$$\frac{dy}{dx} = e^{\frac{x}{2}}\left(\frac{1}{2}\right) = \frac{1}{2}e^{\frac{x}{2}} = \frac{\sqrt{e^x}}{2}$$

---

8) $g(t) = \sqrt{t^3-t} = (t^3-t)^{\frac{1}{2}}$

$$\frac{dg}{dt} = \frac{1}{2}(t^3-t)^{-\frac{1}{2}}(3t^2-1) = \frac{3t^2-1}{2\sqrt{t^3-t}}$$

---

10) $F(x) = (x^2-x+1)^3$

$$\frac{dF}{dx} = 3(x^2-x+1)^2(2x-1) = 3(2x-1)(x^2-x+1)^2$$

---

12) $y = \sqrt[3]{x+e^x} = (x+e^x)^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}(x+e^x)^{-\frac{2}{3}}(1+e^x(1)) = \frac{1+e^x}{3(\sqrt[3]{x+e^x})^2}$$

14) $q(r) = \dfrac{1}{(3r - 1.5r^3)^2} = (3r - 1.5r^3)^{-2}$

$$\frac{dq}{dr} = -2(3r - 1.5r^3)^{-3}(3 - 4.5r^2) = \frac{-2(3 - 4.5r)}{(3r - 1.5r^3)^3} = \underline{\underline{\frac{9r - 6}{(3r - 1.5r^3)^3}}}$$

---

16) $B(t) = 6 + 2e^{-3t}$

$$\frac{dB}{dt} = [0] + 2[e^{-3t}(-3)] = \underline{\underline{-6e^{-3t} = \frac{-6}{e^{3t}}}}$$

---

18) $y = \sqrt{3}\, x\, e^{-5x} = (\sqrt{3}x)(e^{-5x})$

$$\frac{dy}{dx} = [\sqrt{3}](e^{-5x}) + (\sqrt{3}x)[e^{-5x}(-5)] = \sqrt{3}e^{-5x} - 5\sqrt{3}xe^{-5x}$$

$$= \underline{\underline{e^{-5x}\{\sqrt{3} - 5\sqrt{3}x\} = \frac{\sqrt{3} - 5\sqrt{3}x}{e^{5x}}}}$$

---

20) $f(x) = 3 - 2(5^x)$ revised 10/12/2023

$$\frac{df}{dx} = [0] - 2[(5^x)(\ln 5)(1)] = \underline{\underline{-2(\ln 5)5^x}}$$

using formula

$$\frac{d}{du}(a^u) = (a^u)(\ln a)(u')$$

---

26) $y = 10^{(1-x^2)}$

$$\frac{dy}{dx} = (10^{(1-x^2)})(\ln 10)(-2x) = \underline{\underline{(-2x \ln 10)10^{(1-x^2)}}}$$

---

22) $V = 240.2(0.97^x)$

$$\frac{dV}{dx} = 240.2[(0.97^x)(\ln 0.97)(1)] = \underline{\underline{(240.2 \ln 0.97)(0.97^x)}}$$

24) $h(t) = (t^4-1)^3(t^3+1)^4$

$$\frac{dh}{dt} = \left\{\left[3(t^4-1)^2(4t^3)\right](t^3+1)^4 + (t^4-1)^3\left[4(t^3+1)^3(3t^2)\right]\right\}$$

$$= 12t^2(t^4-1)^2(t^3+1)^3\left\{[t](t^3+1) + (t^4-1)[1]\right\}$$

$$= 12t^2(t^4-1)^2(t^3+1)^3\left\{t^4+t+t^4-1\right\}$$

$$= \underline{\underline{12t^2\left\{2t^4+t-1\right\}(t^4-1)^2(t^3+1)^3}}$$

---

28) $r(t) = 0.5e^{t\sqrt{t+1}} = 0.5e^{(t)(t+1)^{\frac{1}{2}}}$

$$\frac{dr}{dt} = 0.5\left[e^{(t)(t+1)^{\frac{1}{2}}}\left\{[1](t+1)^{\frac{1}{2}} + (t)\left[\tfrac{1}{2}(t+1)^{-\frac{1}{2}}(1)\right]\right\}\right]$$

$$= 0.5e^{t\sqrt{t+1}}\left\{\sqrt{t+1} + \frac{t}{2\sqrt{t+1}}\right\}$$

$$= \frac{1}{2}e^{t\sqrt{t+1}}\left\{\left(\frac{\sqrt{t+1}}{1}\right)\left(\frac{2\sqrt{t+1}}{2\sqrt{t+1}}\right) + \frac{t}{2\sqrt{t+1}}\right\}$$

$$= \frac{1}{2}\left\{\frac{2(t+1)+t}{2\sqrt{t+1}}\right\}e^{t\sqrt{t+1}} = \underline{\underline{\left(\frac{3t+2}{4\sqrt{t+1}}\right)e^{t\sqrt{t+1}}}}$$

30) $G(y) = \left(\frac{y^2}{y+1}\right)^5 = \frac{y^{10}}{(y+1)^5}$

$$\frac{dG}{dy} = \frac{\{[10y^9](y+1)^5 - (y^{10})[5(y+1)^4(1)]\}}{((y+1)^5)^2}$$

$$= \frac{5y^9(y+1)^4\{[2](y+1) - (y)[1]\}}{(y+1)^{10}}$$

$$= \frac{5y^9\{2y+2-y\}}{(y+1)^6} = \underline{\underline{\frac{5y^9(y+2)}{(y+1)^6}}}$$

---

32) $y = \frac{e^{-x}+1}{e^x+1}$

$$\frac{dy}{dx} = \frac{[e^{-x}(-1)](e^x+1) - (e^{-x}+1)[e^x(1)]}{(e^x+1)^2}$$

$$= \frac{\left[\frac{-1}{e^x}\right](e^x+1) - \left(\frac{1}{e^x}+1\right)[e^x]}{(e^x+1)^2}$$

$$= \frac{\left(-1-\frac{1}{e^x}\right) - (1+e^x)}{(e^x+1)^2}$$

$$= \frac{-2-\frac{1}{e^x}-e^x}{(e^x+1)^2} = \underline{\underline{\frac{-2-e^{-x}-e^x}{(e^x+1)^2}}}$$

34) $P(t) = \dfrac{250}{1-0.7e^{0.25t}} = 250\left(1-0.7e^{0.25t}\right)^{-1}$

$$\frac{dP}{dt} = 250\left[-1\left(1-0.7e^{0.25t}\right)^{-2}\left(-0.7e^{0.25t}\right)(0.25)\right]$$

$$= \frac{250(0.25)(0.7e^{0.25t})}{(1-0.7e^{0.25t})^2} = \frac{250\left(\frac{1}{4}\right)\left(\frac{7}{10}e^{0.25t}\right)}{(1-0.7e^{0.25t})^2} = \underline{\underline{\frac{\frac{175}{4}e^{0.25t}}{(1-0.7e^{0.25t})^2}}}$$

---

36) $y = \left(5^{(t^2)}+1\right)^4$

$$\frac{dy}{dt} = 4\left(5^{(t^2)}+1\right)^3\left(\left(5^{(t^2)}\right)(\ln 5)(2t)\right) = \underline{\underline{(8t\ln 5)\left(5^{(t^2)}\right)\left(5^{(t^2)}+1\right)^3}}$$

---

38) $y = \sqrt{x+\sqrt{x}} = \left(x+x^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}\left(x+x^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(1+\frac{1}{2}x^{-\frac{1}{2}}\right) = \underline{\underline{\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}}}$$

---

42) $y = xe^{-x}$ $\qquad = \underline{\underline{\dfrac{1-x}{e^x}}}$

$$y' = \frac{dy}{dx} = [1](e^{-x}) + (x)\left[e^{-x}(-1)\right] = e^{-x} - xe^{-x} = \underline{\underline{(1-x)e^{-x}}}$$

$$y'' = \frac{d^2y}{dx^2} = [-1](e^{-x}) + (1-x)\left[e^{-x}(-1)\right] = -e^{-x} - e^{-x} + xe^{-x}$$

$$= \underline{\underline{xe^{-x} - 2e^{-x}}} = (x-2)e^{-x} = \underline{\underline{\frac{x-2}{e^x}}}$$

48) $y = x^2 - 2^x$ $\quad (3, 1)$

$$\frac{dy}{dx} = [2x] - [(2^x)(\ln 2)(1)]$$

$$= 2x - (\ln 2)(2^x)$$

$$m = \left.\frac{dy}{dx}\right|_{x=3} = 2(3) - (\ln 2)(2^{(3)}) = (6 - 8\ln 2)$$

$$y - (1) = (6 - 8\ln 2)(x - (3))$$

$$y - 1 = (6 - 8\ln 2)x - 3(6 - 8\ln 2)$$

$$y = (6 - 8\ln 2)x - 3(6 - 8\ln 2) + 1$$

$$y = (6 - 8\ln 2)x - 18 + 24\ln 2 + 1$$

$$y = (6 - 8\ln 2)x + (24\ln 2) - 17$$

here $m = (6 - 8\ln 2)$

$b = (24\ln 2) - 17$