3.3

$$\frac{d}{du}\{c\} = [0] = 0 \qquad \frac{d}{du}\{a u^n\} = a\left[n u^{(n-1)}\right] = a n u^{(n-1)}$$

$$\frac{d}{du}\{e^u\} = [e^u(1)] = e^u$$

chain rule "section 3.4"

$$\frac{d}{du}\{f(u)g(u)\} = \left[\frac{df}{du}\right](g(u)) + (f(u))\left[\frac{dg}{du}\right]$$ product rule

$$\frac{d}{du}\left\{\frac{f(u)}{g(u)}\right\} = \frac{\left[\frac{df}{du}\right](g(u)) - (f(u))\left[\frac{dg}{du}\right]}{(g(u))^2}, \quad g(u) \neq 0$$ quotient rule

---

2) $F(x) = \frac{x^2 - 3x^3}{x}$

$$\frac{dF}{dx} = \frac{[2x - 9x^2](x) - (x^2 - 3x^3)[1]}{(x)^2}$$

$$= \frac{(2x^2 - 9x^3) - (x^2 - 3x^2)}{x^2}$$

$$= \frac{x^2 - 6x^3}{x^2} = \frac{x^2(1-6x)}{x^2}$$

$$\frac{dF}{dx} = 1 - 6x$$

quotient rule used

$$F(x) = \frac{x^2 - 3x^3}{x}$$

$$= \frac{x^2}{x} - \frac{3x^3}{x}$$

$$F(x) = x - 3x^2$$

$$\frac{dF}{dx} = [1] - 3[2x]$$

$$= 1 - 6x$$

by simplifying first

this method is quicker but only works nicely on denominators that has 1 term only.

4) $g(x) = \sqrt{x}\, e^{x} = (x^{\frac{1}{2}})(e^{x})$

$$\frac{dg}{dx} = \left[\tfrac{1}{2}x^{-\frac{1}{2}}\right](e^{x}) + (x^{\frac{1}{2}})\left[e^{x}(1)\right]$$

$$= \underline{\underline{\frac{e^{x}}{2\sqrt{x}} + \sqrt{x}\, e^{x}}} = \frac{e^{x}}{2\sqrt{x}} + \left(\frac{\sqrt{x}\, e^{x}}{1}\right)\left(\frac{2\sqrt{x}}{2\sqrt{x}}\right) = \underline{\underline{\frac{e^{x} + 2x e^{x}}{2\sqrt{x}}}}$$

---

6) $y = \dfrac{e^{x}}{1+x}$

$$\frac{dy}{dx} = \frac{\left[e^{x}(1)\right](1+x) - (e^{x})[1]}{(1+x)^{2}} = \frac{e^{x} + x e^{x} - e^{x}}{(1+x)^{2}} = \underline{\underline{\frac{x e^{x}}{(1+x)^{2}}}}$$

---

8) $f(t) = \dfrac{2t}{4+t^{2}}$

$$\frac{df}{dt} = \frac{[2](4+t^{2}) - (2t)[2t]}{(4+t^{2})^{2}} = \frac{8 + 2t^{2} - 4t^{2}}{(4+t^{2})^{2}} = \underline{\underline{\frac{8-2t^{2}}{(4+t^{2})^{2}}}}$$

---

10) $R(t) = (t+e^{t})(3-\sqrt{t}) = (t+e^{t})(3-t^{\frac{1}{2}})$

$$\frac{dR}{dt} = \left[1+e^{t}(1)\right](3-t^{\frac{1}{2}}) + (t+e^{t})\left[\tfrac{-1}{2}t^{-\frac{1}{2}}\right]$$

$$= \left[1+e^{t}\right](3-\sqrt{t}) + (t+e^{t})\left[\frac{-1}{2\sqrt{t}}\right]$$

$$= \left\{3-\sqrt{t}+3e^{t}-\sqrt{t}\, e^{t}\right\} - \frac{t+e^{t}}{2\sqrt{t}}$$

$$= 3-\sqrt{t}+3e^{t}-\sqrt{t}\, e^{t} - \frac{t}{2\sqrt{t}} - \frac{e^{t}}{2\sqrt{t}}$$

$$= 3-\sqrt{t}+3e^{t}-\sqrt{t}\, e^{t} - \frac{\sqrt{t}}{2} - \frac{e^{t}}{2\sqrt{t}} = \underline{\underline{3 - \tfrac{3}{2}\sqrt{t} + 3e^{t} - \sqrt{t}\, e^{t} - \frac{e^{t}}{2\sqrt{t}}}}$$

12) $y = \dfrac{t^3+t}{t^4-2}$

$$\frac{dy}{dx} = \frac{[3t^2+1](t^4-2)-(t^3+t)[4t^3]}{(t^4-2)^2}$$

$$= \frac{(3t^6-6t^2+t^4-2)-(4t^6+4t^4)}{(t^4-2)^2} = \underline{\underline{\frac{-t^6-6t^2-3t^4-2}{(t^4-2)^2}}}$$

---

14) $r = (v+2e^v)\sqrt{v} = v(\sqrt{v}) + 2\sqrt{v}\,e^v = v(v^{\frac{1}{2}}) + 2(v^{\frac{1}{2}})(e^v)$

$$= v^{\frac{3}{2}} + 2(v^{\frac{1}{2}})(e^v) = v^{\frac{3}{2}} + (2v^{\frac{1}{2}})(e^v)$$

$$\frac{dr}{dv} = \left[\frac{3}{2}v^{\frac{1}{2}}\right] + \left\{[v^{-\frac{1}{2}}](e^v) + (2v^{\frac{1}{2}})[e^v(1)]\right\}$$

$$= \underline{\underline{\frac{3}{2}\sqrt{v} + \frac{e^v}{\sqrt{v}} + 2\sqrt{v}\,e^v}}$$

---

16) $y = \dfrac{x}{1-\frac{4}{x}} = \left(\dfrac{\frac{x}{1}}{\frac{1}{1}-\frac{4}{x}}\right)\left(\dfrac{\frac{x}{1}}{\frac{x}{1}}\right) = \dfrac{x^2}{x-4}$ GLCD $= x$

$$\frac{dy}{dx} = \frac{[2x](x-4)-(x^2)[1]}{(x-4)^2} = \frac{(2x^2-8x)-(x^2)}{(x-4)^2} = \underline{\underline{\frac{x^2-8x}{(x-4)^2}}}$$

---

18) $y = \dfrac{x^2}{x-0.5e^x} = \dfrac{x^2}{x-\frac{1}{2}e^x}$

$$\frac{dy}{dx} = \frac{[2x](x-\frac{1}{2}e^x)-(x^2)[1-\frac{1}{2}e^x(1)]}{(x-\frac{1}{2}e^x)^2} = \frac{(2x^2-xe^x)-(x^2-\frac{1}{2}x^2e^x)}{(x-\frac{1}{2}e^x)^2}$$

$$= \underline{\underline{\frac{x^2-xe^x+\frac{1}{2}x^2e^x}{(x-\frac{1}{2}e^x)^2}}}$$

20) $h(v) = \dfrac{1+av^2}{1+bv}$

$$\frac{dh}{dv} = \frac{[2av](1+bv)-(1+av^2)[b]}{(1+bv)^2}$$

$$= \frac{(2av+2abv^2)-(b+abv^2)}{(1+bv)^2} = \underline{\underline{\frac{2av+abv^2-b}{(1+bv)^2}}}$$

---

22) $y = e^x(0.8x^2-x) = e^x\left(\frac{8}{10}x^2-x\right) = e^x\left(\frac{4}{5}x^2-x\right)$, $x=1.5=\frac{3}{2}$

$$\frac{dy}{dx} = [e^x(1)]\left(\frac{4}{5}x^2-x\right)+(e^x)\left[\frac{8}{5}x-1\right]$$

$$= \left(\frac{4}{5}x^2e^x - xe^x\right)+\left(\frac{8}{5}xe^x-e^x\right) = \frac{4}{5}x^2e^x+\frac{3}{5}xe^x-e^x$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{3}{2}} = \frac{4}{5}\left(\frac{3}{2}\right)^2e^{\left(\frac{3}{2}\right)}+\frac{3}{5}\left(\frac{3}{2}\right)e^{\left(\frac{3}{2}\right)}-e^{\left(\frac{3}{2}\right)}$$

$$= \frac{9}{5}e^{\left(\frac{3}{2}\right)}+\frac{9}{10}e^{\left(\frac{3}{2}\right)}-e^{\left(\frac{3}{2}\right)}$$

$$= \frac{18}{10}e^{\left(\frac{3}{2}\right)}+\frac{9}{10}e^{\left(\frac{3}{2}\right)}-\frac{10}{10}e^{\left(\frac{3}{2}\right)} = \frac{17}{10}e^{\left(\frac{3}{2}\right)} = \underline{\underline{\frac{17}{10}\left(\sqrt{e}\right)^3}}$$

---

24) $y = \dfrac{\sqrt{x}}{x+1} = \dfrac{x^{\frac{1}{2}}}{x+1}$ $(4, 0.4)$

$$\frac{dy}{dx} = \frac{\left[\frac{1}{2}x^{-\frac{1}{2}}\right](x+1)-\left(x^{\frac{1}{2}}\right)[1]}{(x+1)^2} = \frac{\frac{x+1}{2\sqrt{x}}-\frac{\sqrt{x}}{1}\left(\frac{2\sqrt{x}}{2\sqrt{x}}\right)}{(x+1)^2}$$

$$= \frac{\frac{(x+1)-2x}{2\sqrt{x}}}{(x+1)^2} = \frac{\frac{1-x}{2\sqrt{x}}}{(x+1)^2} = \left(\frac{1-x}{2\sqrt{x}}\right)\left(\frac{1}{(x+1)^2}\right) = \frac{1-x}{2\sqrt{x}\,(x+1)^2}$$

24) continued

$$m = \left.\frac{dy}{dx}\right|_{x=4} = \frac{1-(4)}{2\sqrt{(4)}\,((4)+1)^2} = \frac{-3}{2(2)(5)^2} = \frac{-3}{100}$$

$$y - (0.4) = \frac{-3}{100}(x-(4))$$

$$y - \frac{40}{100} = \frac{-3}{100}x + \frac{12}{100}$$

$$y = \frac{-3}{100}x + \frac{52}{100} = \frac{-3}{100}x + \frac{26}{50} = \underline{\underline{\frac{-3}{100}x + \frac{13}{25}}}$$

---

36) $A(r) = \dfrac{2r}{r^2+1}$ $\qquad A''(r) = \dfrac{d^2A}{dr^2} = ?$

$$\frac{dA}{dr} = \frac{[2](r^2+1)-(2r)[2r]}{(r^2+1)^2} = \frac{(2r^2+2)-(4r^2)}{(r^2+1)^2} = \underline{\frac{2-2r^2}{(r^2+1)^2}}$$

$$\frac{d^2A}{dr^2} = \frac{\{[-4r]((r^2+1)^2) - (2-2r^2)[2(r^2+1)'(2r)]\}}{((r^2+1)^2)^2}$$

used chain rule (section 3.4) for 2nd derivative

$$= \frac{(r^2+1)\{[-4r](r^2+1) - (2-2r^2)[4r]\}}{(r^2+1)^4}$$

$$= \frac{(-4r^3-4r)-(8r-8r^3)}{(r^2+1)^3}$$

$$= \frac{-4r^3-4r-8r+8r^3}{(r^2+1)^3}$$

$$\underline{\underline{\frac{d^2A}{dr^2} = \frac{4r^3-12r}{(r^2+1)^3}}}$$

40) $f(x) = \frac{g(x)}{e^x}$; $g(1) = -1$ and $g'(1) = \left.\frac{dg}{dx}\right|_{x=1} = 3$

$$\frac{df}{dx} = \frac{\left[\frac{dg}{dx}\right](e^x) - (g(x))\left[e^x(1)\right]}{(e^x)^2} = \frac{e^x\left\{\left[\frac{dg}{dx}\right](1) - (g(x))[1]\right\}}{(e^x)^2}$$

$$= \frac{\frac{dg}{dx} - g(x)}{e^x}$$

$$f'(1) = \left.\frac{df}{dx}\right|_{x=1} = \frac{\left.\frac{dg}{dx}\right|_{x=1} - g(1)}{e^{(1)}} = \frac{(3)-(-1)}{e^{(1)}} = \underline{\underline{\frac{4}{e}}}$$

---

42) $P(x) = F(x)G(x)$ $\quad P'(x) = \frac{dP}{dx} = \left[\frac{dF}{dx}\right](G(x)) + (F(x))\left[\frac{dG}{dx}\right]$

a) $P'(2) = \left.\frac{dP}{dx}\right|_{x=2} = ?$ from graph $F(2) = 3$ $\quad F'(2) = \left.\frac{dF}{dx}\right|_{x=2} = 0$

$G(2) = 2$ $\quad G'(2) = \left.\frac{dG}{dx}\right|_{x=2} = \frac{1}{2}$

$$P'(2) = \left.\frac{dP}{dx}\right|_{x=2} = \left[\left.\frac{dF}{dx}\right|_{x=2}\right](G(2)) + (F(2))\left[\left.\frac{dG}{dx}\right|_{x=2}\right] = [0](2) + (3)\left[\frac{1}{2}\right] = \underline{\underline{\frac{3}{2}}}$$

$Q(x) = \frac{F(x)}{G(x)}$ $\quad Q'(x) = \frac{dQ}{dx} = \frac{\left[\frac{dF}{dx}\right](G(x)) - (F(x))\left[\frac{dG}{dx}\right]}{(G(x))^2}$

b) $Q'(7) = \left.\frac{dQ}{dx}\right|_{x=7} = ?$ from graph $F(7) = 5$ $\quad F'(7) = \left.\frac{dF}{dx}\right|_{x=7} = \frac{1}{4}$

$G(7) = 1$ $\quad G'(7)\ \left.\frac{dG}{dx}\right|_{x=7} = \frac{-2}{3}$

$$Q'(7) = \left.\frac{dQ}{dx}\right|_{x=7} = \frac{\left[\left.\frac{dF}{dx}\right|_{x=7}\right](G(7)) - (F(7))\left[\left.\frac{dG}{dx}\right|_{x=7}\right]}{(G(7))^2}$$

$$= \frac{\left[\frac{1}{4}\right](1) - (5)\left[\frac{-2}{3}\right]}{(1)^2} = \frac{\frac{1}{4} + \frac{10}{3}}{1} = \frac{1}{4} + \frac{10}{3} = \frac{1}{4}\left(\frac{3}{3}\right) + \frac{10}{3}\left(\frac{4}{4}\right)$$

$$= \frac{3}{12} + \frac{40}{12} = \underline{\underline{\frac{43}{12}}}$$