3.2

4) $C(q) = 73 + 0.6q + 0.004q^2$ thousand dollars per q tons of copper

a) fixed cost: $C(0) = 73 + 0.6(0) + 0.004(0)^2 = \underline{73 \text{ thousand dollars}}$

b) average cost: $\frac{C(q)}{q} = \frac{73 + 0.6q + 0.004q^2}{q} = \frac{73}{q} + 0.6 + 0.004q$

$q = 150$: $\frac{C(150)}{150} = \frac{73}{(150)} + 0.6 + 0.004(150) = \frac{73}{150} + 0.6 + 0.6 = \frac{73}{150} + 1.2$

$= \frac{73}{150} + \frac{12}{10} = \frac{73}{150} + \left(\frac{12}{10}\right)\left(\frac{15}{15}\right) = \frac{73}{150} + \frac{180}{150} = \underline{\underline{\frac{253}{150}}}$

c) marginal cost: $C'(q) = \frac{dC}{dq} = [0] + 0.6[1] + 0.004[2q]$

$= 0.6 + 0.008q$

d) $C'(150) = \left.\frac{dC}{dq}\right|_{q=150} = 0.6 + 0.008(150) = 0.6 + 1.2 = 1.8$

$= \$1.8$ thousand dollars per ton of copper

$C(150) = 73 + 0.6(150) + 0.004(150)^2 = 73 + 90 + 90 = 253$

$C(151) = 73 + 0.6(151) + 0.004(151)^2 = 73 + 90.6 + 91.204 = 254.804$

actual cost of extracting 151st tons of copper:

$C(151) - C(150) = (254.804) - (253) = 1.804$

$= \$1.804$ thousand dollar per ton of copper

marginal cost for 150 tons of copper is similar to actual cost of extracting 151st tons of copper.

6) $C(q) = 84 + 0.16q - 0.0006q^2 + 0.000003q^3$

a) $C'(q) = \frac{dC}{dq} = [0] + 0.16[1] - 0.0006[2q] + 0.000003[3q^2]$

marginal cost

$= 0.16 - 0.0012q + 0.000009q^2$

$C'(100) = \frac{dC}{dq}\Big|_{q=100} = 0.16 - 0.0012(100) + 0.000009(100)^2$

$= 0.16 - 0.12 + 0.09 = 0.13$ dollars per item

b) $C(100) = 84 + 0.16(100) - 0.0006(100)^2 + 0.000003(100)^3$

$= 84 + 16 - 6 + 3 = 97$

$C(101) = 84 + 0.16(101) - 0.0006(101)^2 + 0.000003(101)^2$

$= 84 + 16.16 - 6.1206 + 3.090903$

$= 97.130303$

actual cost of producing 101st item:

$C(101) - C(100) = (97.130303) - (97) = 0.130303$ dollars per item

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10) $g(125) = 5250$, $g'(125) = -13.8$

linear approximation {Euler's method} $f(a+\delta x) \approx f(a) + f'(a)\delta x$

a) $g(127.4) = g(\overset{a}{125} + \overset{\delta x}{2.4}) \approx g(125) + g'(125)(2.4)$

$= (5250) + (-13.8)(2.4)$

$= 5250 - 33.12 = \underline{5216.88}$

b) $g(121) = g(\overset{a}{125} + \overset{\delta x}{(-4)}) \approx g(125) + g'(125)(-4) = (5250) + (-13.8)(-4)$

$= 5250 + 55.2 =$

$= \underline{5305.2}$

12) this is a linear approximation problem.

v: gallons of drinking water

$C(v)$: in dollars

$$C'(200000) = \frac{dC}{dv}\Big|_{v=200000} = 0.26$$

additional 3000 gallons of water $\Longrightarrow \Delta x = 3000$

200000 gallons have been processed $\Longrightarrow a = 200000$

$$C(a+\Delta x) \simeq C(a) + \underbrace{C'(a)\Delta x}_{\text{we need to find this part}}$$

$$C'(a)\Delta x = (0.26)(3000) = (26)(30) = 780$$

about \$780 is needed in order to purify an additional 3000 gallons of water.