Math 201 GH — **EXAM #1** — March 16, 2016

Write clearly and cross-out work not to be graded.

1. (10 pts.) Find the following limits, or state that they do not exist (DNE) (show a calculation or give an explanation):

   (a)

   $$\lim_{x\to\infty} \frac{x^3-1}{2x^3-x+1}$$

   (b)

   $$\lim_{x\to -4} \frac{x^2-16}{x+4}$$

2. (20 pts.) Find the derivative y' and simplify:

   (a) $(x^5-2\sqrt{x})^{10}$

   (b)

   $$y=\frac{2x^2-1}{3x+5}$$

   (c) $y=\sin^3(x^2)$

   (d) implicitly, assuming $y^5+x^2y^3=1+x^4y$

3. (10 pts.) Let

   $$f(x)=\begin{cases} 1 & x=0 \\ \frac{\sin x}{x} & x\neq 0 \end{cases}$$

   Determine if f is continuous at each of the following values and **explain** why or why not.

   (a) $x=0$:

   (b) $x=2\pi$:

4. (15 pts.) Let $f(x)=x^{1/3}$.

   (a) Find the differential df:

   (b) Find the linear approximation $L(x)$ to $f(x)$ near $x=8$:

   (c) Use the linear approximation to estimate $(7.9)^{1/3}$:

5. (15 pts.) The displacement (in meters) of a particle moving in a straight line is given by $s(t) = t^2 - t + 6$, where time t is measured in seconds.

   (a) Find the instantaneous velocity $v(t)$ (include units):

   (b) For what values of $t \geq 0$ is the particle moving to the left, and for which is it moving to the right?

   (c) What is the location of the particle when it turns around?

6. (15 pts.)Two cars start moving from the same point. One travels south at 1.5 mi/hr and the other travels west at 2 mi/hr. At what rate is the distance between the cars increasing two hours later?

7. (a) (10 pts.) Use the **definition** of derivative, NOT the rules of differentiation, to find the derivative $f'(x)$ if $f(x) = \frac{1}{x-2}$:

   (b) (5 pts.) Using the $\epsilon - \delta$ definition of limit, **prove**

$$\lim_{x \to 1}(x^2 - 2x) = -1$$

   (recall given $\epsilon > 0$, which bounds $f(x)$ near the limit, you must find the appropriate value of δ, which bounds x near 1):