## Section 4.1 and 4.2

The graph of General Exponential function with base $b$ is defined for all real numbers $x$ by

$$
y=(a) b^{(x-H)}+V
$$

This graph has a Horizontal Asymptote: $y=V$ This value is also a Vertical Shift.
$a$ is the stretching factor
$H$ is Horizontal Shift
Domain: $(-\infty, \infty)$
Range: if $a>0,(V, \infty)$; if $a<0,(-\infty, V)$
This is also true if the base $b=e$ which is a natural base covered in section 4.2 and it will look like:


$$
y=(a) e^{(x-H)}+V
$$

## Section 4.3

The graph of General Logarithmic function with base $b$ :

$$
y=(a) \log _{b}(x-H)+V
$$

This graph has a Vertical Asymptote:
solve $x-H=0$ for $x$.
$a$ is the stretching factor
$V$ is Vertical Shift
Domain: solve $x-H>0$ for $x$.
Range: $(-\infty, \infty)$

This is also true if the base $b=e$ which is a natural base and it will look like:


$$
y=(a) \ln (x-H)+V
$$

| Conversion formula: |  |  |
| :---: | :---: | :---: |
| $b^{p}=N$ | $\Leftrightarrow \quad p=\log _{b} N$ |  |
| $(10)^{p}=N$ | $\Leftrightarrow$ | $p=\log N$ |
| $e^{p}=N$ | $\Leftrightarrow$ | $p=\ln N$ |


| Properties of Logarithms: |  |
| :---: | :---: |
| $\log _{b} 1=0$ | $\ln 1=0$ |
| $\log _{b}(b)=1$ | $\ln (e)=1$ |
| $\log _{b}\left(b^{x}\right)=x$ | $\ln \left(e^{x}\right)=x$ |
| $b^{\log _{b} x}=x$ | $e^{\ln x}=x$ |

## Section 4.4

Laws of Logarithms

| $\log _{b}(A B)=\log _{b} A+\log _{b} B$ | $\ln (A B)=\ln A+\ln B$ |
| :--- | :--- |
| $\log _{b}\left(\frac{A}{B}\right)=\log _{b} A-\log _{b} B$ | $\ln \left(\frac{A}{B}\right)=\ln A-\ln B$ |
| $\log _{b}\left(A^{p}\right)=p \log _{b} A$ | $\ln \left(A^{p}\right)=p \ln A$ |

## Change of Base Formula

$$
\log _{b} A=\frac{\ln A}{\ln b}=\frac{\log A}{\log b}
$$

## Section 4.6

## Exponential Growth (Doubling Time)

If the initial size of a population is $n_{0}$ and the doubling time is $a$, then the size of the population at time $t$ is

$$
n(t)=n_{0} 2^{t / a}
$$

Where $a$ and $t$ are measured in the same time units (minutes, hours, days, years, and so on).

## Exponential Growth (Relative Growth Rate)

A population that experiences exponential growth increases according to the model

$$
n(t)=n_{0} e^{r t} \quad \text { or } \quad P(t)=P_{0} e^{r t}
$$

where: $\quad n(t)=P(t)=$ population at time $t$
$n_{0}=P_{0}=$ initial size of the population
$r=$ relative rate of growth (expressed as a proportion of the population)
$t=$ time

