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section 7.2

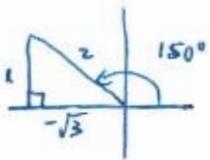
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2) the subtraction Formula for Cosine.

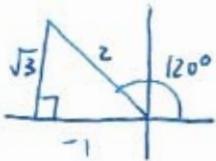
$$\cos(x-y) = \cos x \cos y - \sin x \sin y$$

$$\begin{aligned} 4) \sin 15^\circ &= \sin(45^\circ - 30^\circ) = \sin(45^\circ) \cos(30^\circ) - \cos(45^\circ) \sin(30^\circ) \\ &= \left(\frac{1}{\sqrt{2}}\right) \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} 6) \cos 195^\circ &= \cos(150^\circ + 45^\circ) = \cos(150^\circ) \cos(45^\circ) - \sin(150^\circ) \sin(45^\circ) \\ &= \left(\frac{-\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right) = \frac{-\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{-\sqrt{3}-1}{2\sqrt{2}} = \frac{-(\sqrt{3}+1)}{2\sqrt{2}} \end{aligned}$$



$$8) \tan 165^\circ = \tan(120^\circ + 45^\circ) = \frac{\sin(120^\circ + 45^\circ)}{\cos(120^\circ + 45^\circ)}$$



$$\begin{aligned} &= \frac{\sin(120^\circ) \cos(45^\circ) + \cos(120^\circ) \sin(45^\circ)}{\cos(120^\circ) \cos(45^\circ) - \sin(120^\circ) \sin(45^\circ)} \\ &= \frac{\left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right)} = \frac{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \end{aligned}$$

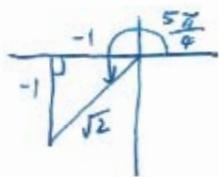
$$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) \left(\frac{2\sqrt{2}}{\sqrt{3}+1}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

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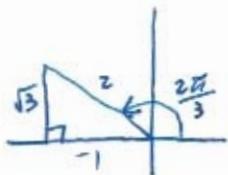
$$10) \cos\left(\frac{17\pi}{12}\right) = \cos\left(\frac{15\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$$



$$= \cos\left(\frac{5\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{5\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{-\sqrt{3}+1}{2\sqrt{2}} = \underline{\underline{\frac{1-\sqrt{3}}{2\sqrt{2}}}}$$

$$12) \sin\left(-\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{2\pi}{3}\right)$$



$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{-1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \underline{\underline{\frac{-1-\sqrt{3}}{2\sqrt{2}}}}$$

$$14) \tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)}$$

$$= \frac{\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}}$$

$$= \frac{\frac{1+\sqrt{3}}{2\sqrt{2}}}{\frac{1-\sqrt{3}}{2\sqrt{2}}} = \left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{2\sqrt{2}}{1-\sqrt{3}}\right) = \underline{\underline{\frac{1+\sqrt{3}}{1-\sqrt{3}}}}$$

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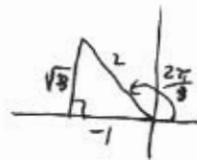
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$$16) \cos 10^\circ \cos 80^\circ - \sin 10^\circ \sin 80^\circ = \cos(10^\circ + 80^\circ) = \cos 90^\circ = \underline{0}$$

$$18) \text{ Addition formula for Tangent } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\tan \frac{\pi}{18} + \tan \frac{\pi}{9}}{1 - \tan \frac{\pi}{18} \tan \frac{\pi}{9}} = \tan\left(\frac{\pi}{18} + \frac{\pi}{9}\right) = \tan\left(\frac{\pi}{18} + \frac{2\pi}{18}\right) = \tan\left(\frac{3\pi}{18}\right) \\ = \tan\left(\frac{\pi}{6}\right) = \underline{\underline{\frac{1}{\sqrt{3}}}}$$

$$20) \cos \frac{13\pi}{15} \cos\left(-\frac{\pi}{5}\right) - \sin \frac{13\pi}{15} \sin\left(-\frac{\pi}{5}\right) = \cos\left(\frac{13\pi}{15} + \left(-\frac{\pi}{5}\right)\right) \\ = \cos\left(\frac{13\pi}{15} - \frac{3\pi}{15}\right) = \cos\left(\frac{10\pi}{15}\right) = \cos\left(\frac{2\pi}{3}\right) = \underline{\underline{-\frac{1}{2}}}$$



$$22) \cot\left(\frac{\pi}{2} - x\right) = \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} = \frac{\cos\left(\frac{\pi}{2}\right) \cos x + \sin\left(\frac{\pi}{2}\right) \sin x}{\sin\left(\frac{\pi}{2}\right) \cos x - \cos\left(\frac{\pi}{2}\right) \sin x} \\ = \frac{(0) \cos x + (1) \sin x}{(1) \cos x - (0) \sin x} = \frac{\sin x}{\cos x} = \underline{\underline{\tan x}}$$

$$24) \csc\left(\frac{\pi}{2} - u\right) = \frac{1}{\sin\left(\frac{\pi}{2} - u\right)} = \frac{1}{\sin\left(\frac{\pi}{2}\right) \cos u - \cos\left(\frac{\pi}{2}\right) \sin u} \\ = \frac{1}{(1) \cos u - (0) \sin u} = \frac{1}{\cos u} = \underline{\underline{\sec u}}$$

$$26) \cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) + \sin x \sin\left(\frac{\pi}{2}\right)$$

$$= \cos x (0) + \sin x (1) = \underline{\underline{\sin x}}$$

$$28) \cos(x - \pi) = \cos x \cos(\pi) + \sin x \sin(\pi)$$

$$= \cos x (-1) + \sin x (0) = \underline{\underline{-\cos x}}$$

$$30) \tan\left(x - \frac{\pi}{2}\right) = \frac{\sin\left(x - \frac{\pi}{2}\right)}{\cos\left(x - \frac{\pi}{2}\right)} = \frac{\sin x \cos\left(\frac{\pi}{2}\right) - \cos x \sin\left(\frac{\pi}{2}\right)}{\cos x \cos\left(\frac{\pi}{2}\right) + \sin x \sin\left(\frac{\pi}{2}\right)}$$

$$= \frac{\sin x (0) - \cos x (1)}{\cos x (0) + \sin x (1)} = \frac{-\cos x}{\sin x} = \underline{\underline{-\cot x}}$$

$$32) \cos\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{6}\right)$$

$$= \left\{ \cos x \cos\left(\frac{\pi}{3}\right) - \sin x \sin\left(\frac{\pi}{3}\right) \right\} + \left\{ \sin x \cos\left(\frac{\pi}{6}\right) - \cos x \sin\left(\frac{\pi}{6}\right) \right\}$$

$$= \left\{ \cos x \left(\frac{1}{2}\right) - \sin x \left(\frac{\sqrt{3}}{2}\right) \right\} + \left\{ \sin x \left(\frac{\sqrt{3}}{2}\right) - \cos x \left(\frac{1}{2}\right) \right\}$$

$$= \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x + \frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x = \underline{\underline{0}}$$

34) Subtraction formula for Tangent $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - \tan\left(\frac{\pi}{4}\right)}{1 + \tan x \tan\left(\frac{\pi}{4}\right)} = \frac{\tan x - (1)}{1 + \tan x (1)} = \frac{\tan x - 1}{1 + \tan x}$$

$$= \underline{\underline{\frac{\tan x - 1}{\tan x + 1}}}$$

$$36) \cos(x+y) + \cos(x-y)$$

$$= \{ \cos x \cos y - \sin x \sin y \} + \{ \cos x \cos y + \sin x \sin y \}$$

$$= \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$$

$$= \underline{\underline{2 \cos x \cos y}}$$

$$38) \cot(x+y)$$

$$\frac{\cos(x+y)}{\sin(x+y)}$$

$$\frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}$$



$$\frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y} =$$

$$\frac{\cot x \cot y - 1}{\cot y + \cot x}$$

$$\frac{\left(\frac{\cos x}{\sin x}\right)\left(\frac{\cos y}{\sin y}\right) - 1}{\left(\frac{\cos y}{\sin y}\right) + \left(\frac{\cos x}{\sin x}\right)}$$

$$\text{GLCD} = \sin x \sin y$$

$$\left(\frac{\frac{\cos x \cos y}{\sin x \sin y} - 1}{\frac{\cos y}{\sin y} + \frac{\cos x}{\sin x}}\right) \left(\frac{\frac{\sin x \sin y}{1}}{\frac{\sin x \sin y}{1}}\right)$$

$$\frac{\cos x \cos y - \sin x \sin y}{\cos y \sin x + \cos x \sin y}$$

$$\frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}$$

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$$40) 1 - \tan x \tan y$$

$$\frac{\cos(x+y)}{\cos x \cos y}$$

$$1 - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)$$

$$\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$$

$$1 - \frac{\sin x \sin y}{\cos x \cos y}$$

$$\frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x \sin y}{\cos x \cos y}$$

$$1 - \frac{\sin x \sin y}{\cos x \cos y} = 1 - \frac{\sin x \sin y}{\cos x \cos y}$$

$$42) \frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \frac{\{\sin x \cos y + \cos x \sin y\} - \{\sin x \cos y - \cos x \sin y\}}{\{\cos x \cos y - \sin x \sin y\} + \{\cos x \cos y + \sin x \sin y\}}$$

$$= \frac{\sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y}$$

$$= \frac{2 \cos x \sin y}{2 \cos x \cos y} = \frac{\sin y}{\cos y} = \underline{\underline{\tan y}}$$

$$44) \cos(x+y) \cos y + \sin(x+y) \sin y$$

$$= \{ \cos x \cos y - \sin x \sin y \} \cos y + \{ \sin x \cos y + \cos x \sin y \} \sin y$$

$$= \cos x \cos^2 y - \sin x \sin y \cos y + \sin x \sin y \cos y + \cos x \sin^2 y$$

$$= \cos x \cos^2 y + \cos x \sin^2 y = \cos x [\cos^2 y + \sin^2 y] = \cos x [1] = \underline{\underline{\cos x}}$$

$$46) \tan(x-y) + \tan(y-z) + \tan(z-x) \stackrel{?}{=} \tan(x-y) \tan(y-z) \tan(z-x)$$

$$\text{let } A = (x-y) \text{ and } B = (y-z)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan((x-y) + (y-z)) = \frac{\tan(x-y) + \tan(y-z)}{1 - \tan(x-y) \tan(y-z)}$$

$$\tan(x-z) = \frac{\tan(x-y) + \tan(y-z)}{1 - \tan(x-y) \tan(y-z)}$$

$$-\tan(z-x) = \frac{\tan(x-y) + \tan(y-z)}{(1 - \tan(x-y) \tan(y-z))}$$

$$-\tan(z-x) [1 - \tan(x-y) \tan(y-z)] = \tan(x-y) + \tan(y-z)$$

$$\tan(x-y) + \tan(y-z) + \tan(z-x) = \{ \tan(x-y) + \tan(y-z) \} + \tan(z-x)$$

$$= \{ -\tan(z-x) [1 - \tan(x-y) \tan(y-z)] \} + \tan(z-x)$$

$$= -\tan(z-x) + \tan(x-y) \tan(y-z) \tan(z-x) + \tan(z-x)$$

$$= -\tan(z-x) + \tan(z-x) + \tan(x-y) \tan(y-z) \tan(z-x)$$

$$= 0 + \tan(x-y) \tan(y-z) \tan(z-x) = \underline{\underline{\tan(x-y) \tan(y-z) \tan(z-x)}}$$

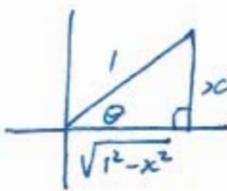
$$\begin{aligned} & \tan(x-z) \\ &= \frac{\tan x - \tan z}{1 + \tan x \tan z} \end{aligned}$$

$$= \frac{-\tan z + \tan x}{1 + \tan x \tan z}$$

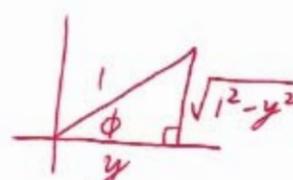
$$= \frac{-(\tan z - \tan x)}{1 + \tan z \tan x}$$

$$= -\tan(z-x)$$

48) $\theta = \sin^{-1} x$
 \downarrow
 $\sin \theta = x = \frac{x}{1}$



$\phi = \cos^{-1} y$
 \downarrow
 $\cos \phi = y = \frac{y}{1}$



$$\tan(\sin^{-1} x + \cos^{-1} y) = \tan(\theta + \phi)$$

$$= \frac{\tan(\theta) + \tan(\phi)}{1 - \tan(\theta)\tan(\phi)} = \frac{\left(\frac{x}{\sqrt{1-x^2}}\right) + \left(\frac{\sqrt{1-y^2}}{y}\right)}{1 - \left(\frac{x}{\sqrt{1-x^2}}\right)\left(\frac{\sqrt{1-y^2}}{y}\right)} \quad \text{GLCD} = y\sqrt{1-x^2}$$

$$= \frac{\left(\frac{x}{\sqrt{1-x^2}} + \frac{\sqrt{1-y^2}}{y}\right) \left(\frac{y\sqrt{1-x^2}}{1}\right)}{\left(1 - \frac{x\sqrt{1-y^2}}{y\sqrt{1-x^2}}\right) \left(\frac{y\sqrt{1-x^2}}{1}\right)} = \frac{xy + \sqrt{1-x^2}\sqrt{1-y^2}}{y\sqrt{1-x^2} - x\sqrt{1-y^2}}$$

50) see triangles for θ and ϕ from exercise 48.

$$\sin(\sin^{-1} x + \cos^{-1} y) = \sin(\theta + \phi)$$

$$= \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$= \left(\frac{x}{1}\right)\left(\frac{y}{1}\right) + \left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{\sqrt{1-y^2}}{1}\right)$$

$$= \underline{\underline{xy + \sqrt{1-x^2}\sqrt{1-y^2}}}$$

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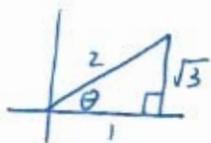
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$$52) \quad \theta = \sin^{-1} \frac{\sqrt{3}}{2}$$

$$\Downarrow$$

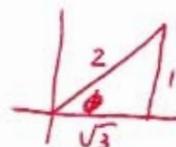
$$\sin \theta = \frac{\sqrt{3}}{2}$$



$$\phi = \cot^{-1} \sqrt{3}$$

$$\Downarrow$$

$$\cot \phi = \sqrt{3} \Rightarrow \tan \phi = \frac{1}{\sqrt{3}}$$



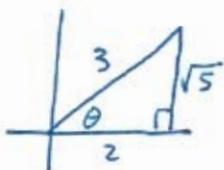
$$\cos \left(\sin^{-1} \frac{\sqrt{3}}{2} + \cot^{-1} \sqrt{3} \right) = \cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi)$$

$$= \left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right) - \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \underline{\underline{0}}$$

$$54) \quad \theta = \cos^{-1} \frac{2}{3}$$

$$\Downarrow$$

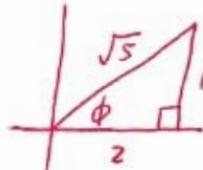
$$\cos \theta = \frac{2}{3}$$



$$\phi = \tan^{-1} \frac{1}{2}$$

$$\Downarrow$$

$$\tan \phi = \frac{1}{2}$$



$$\sin \left(\cos^{-1} \frac{2}{3} - \tan^{-1} \frac{1}{2} \right) = \sin(\theta - \phi)$$

$$= \sin(\theta) \cos(\phi) - \cos(\theta) \sin(\phi)$$

$$= \left(\frac{\sqrt{5}}{3} \right) \left(\frac{2}{\sqrt{5}} \right) - \left(\frac{2}{3} \right) \left(\frac{1}{\sqrt{5}} \right)$$

$$= \underline{\underline{\frac{2\sqrt{5} - 2}{3\sqrt{5}}}}$$