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section 4.3

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2) base 9. So $f(9)=1$, $f(1)=0$, $f\left(\frac{1}{9}\right)=-1$, $f(81)=2$,
and $f(3)=\frac{1}{2}$

4-a) $f(x) = \log_2 x$ Graph III

4-b) $f(x) = \log_2(-x)$ Graph II

4-c) $f(x) = -\log_2(x)$ Graph I

4-d) $f(x) = -\log_2(-x)$ Graph IV

5) $f(x) = \ln(x-1) \Rightarrow x-1=0 \Rightarrow x=1$

has the vertical asymptote $x=1$

8) $4^3 = 64 \Rightarrow \log_4 64 = 3$; $\log_4\left(\frac{1}{2}\right) = -\frac{1}{2} \Rightarrow 4^{-\frac{1}{2}} = \frac{1}{2}$

$\log_4 2 = \frac{1}{2} \Rightarrow 4^{\frac{1}{2}} = 2$

$4^{\frac{3}{2}} = 8 \Rightarrow \log_4 8 = \frac{3}{2}$; $4^{-\frac{5}{2}} = \frac{1}{32} \Rightarrow \log_4\left(\frac{1}{32}\right) = -\frac{5}{2}$

$\log_4\left(\frac{1}{16}\right) = -2 \Rightarrow 4^{-2} = \frac{1}{16}$

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(2)

$$10-a) \log_5 \left(\frac{1}{5}\right) = -1 \Rightarrow 5^{-1} = \frac{1}{5} \quad | \quad 10-b) \log_4 64 = 3 \Rightarrow 4^3 = 64$$

$$12-a) \log_5 \left(\frac{1}{125}\right) = -3 \Rightarrow 5^{-3} = \frac{1}{125} \quad | \quad 12-b) \log_8 4 = \frac{2}{3} \Rightarrow 8^{\frac{2}{3}} = 4$$

$$14-a) \log_6 3 = 1 \Rightarrow 6^1 = 3 \quad | \quad 14-b) \log_{10} 3 = 2t \Rightarrow 10^{2t} = 3$$

$$16-a) \ln(x+1) = 2 \Rightarrow e^2 = (x+1) \quad | \quad 16-b) \ln(x-1) = 4 \Rightarrow e^4 = (x-1)$$

$$18-a) 6^2 = 36 \Rightarrow \log_6 36 = 2 \quad | \quad 18-b) 10^{-1} = \frac{1}{10} \Rightarrow \log_{10} \left(\frac{1}{10}\right) = -1 \\ \text{or } \log \left(\frac{1}{10}\right) = -1$$

$$20-a) 4^{-\frac{3}{2}} = 0.125 \Rightarrow \log_4 (0.125) = -\frac{3}{2} \quad | \quad 20-b) 7^{\frac{3}{3}} = 343 \Rightarrow \log_7 343 = 3$$

$$22-a) 3^{2x} = 10 \Rightarrow \log_3 10 = 2x \quad | \quad 22-b) 10^{-4x} = 0.1 \Rightarrow \log_{10} (0.1) = -4x \\ \text{or } \log (0.1) = -4x$$

$$24-a) e^{x+1} = 0.5 \Rightarrow \ln 0.5 = (x+1) \quad | \quad 24-b) e^{0.5x} = t \Rightarrow \ln t = 0.5x$$

$$26-a) \log_3 3^7 = x \quad | \quad 26-b) \log_4 64 = x \quad | \quad 26-c) \log_5 125 = x \\ \Downarrow \qquad \qquad \Downarrow \qquad \qquad \Downarrow$$

$$3^x = 3^7$$

$$4^x = 64$$

$$4^x = 4^3$$

$$5^x = 125$$

$$5^x = 5^3$$

$$\underline{\underline{\text{so } x=7}}$$

$$\underline{\underline{\text{so } x=3}}$$

$$\underline{\underline{\text{so } x=3}}$$

28-a) $\log_2 32 = x$ \Downarrow $2^x = 32$ $2^x = 2^5$ $\text{so } \underline{\underline{x=5}}$	28-b) $\log_8 8^{17} = x$ \Downarrow $8^x = 8^{17}$ $\text{so } \underline{\underline{x=17}}$	28-c) $\log_6 1 = x$ \Downarrow $6^x = 1$ $6^x = 6^0$ $\text{so } \underline{\underline{x=0}}$
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30-a) $\log_5 125 = x$ \Downarrow $5^x = 125$ $5^x = 5^3$ $\text{so } \underline{\underline{x=3}}$	30-b) $\log_{49} 7 = x$ \Downarrow $49^x = 7$ $49^x = \sqrt{49} = 49^{\frac{1}{2}}$ $\text{so } \underline{\underline{x=\frac{1}{2}}}$	30-c) $\log_9 \sqrt{3} = x$ \Downarrow $9^x = \sqrt{3}$ $9^x = 3^{\frac{1}{2}}$ $9^x = (\sqrt{9})^{\frac{1}{2}}$ $9^x = (9^{\frac{1}{2}})^{\frac{1}{2}} = 9^{\frac{1}{4}}$ $\text{so } \underline{\underline{x=\frac{1}{4}}}$
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32-a) $e^{\ln \sqrt{3}} = \sqrt{3}$	32-b) $e^{\ln(\frac{1}{\pi})} = \frac{1}{\pi}$	32-c) $10^{\log 13} = 13$
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since the function
 e^x and $\ln x$ are
inverse functions

34-a) $\log_4 \sqrt{2} = x$ \Downarrow $4^x = \sqrt{2}$ $4^x = 2^{\frac{1}{2}}$ $4^x = (\sqrt{4})^{\frac{1}{2}}$ $4^x = (4^{\frac{1}{2}})^{\frac{1}{2}} = 4^{\frac{1}{4}}$ $\text{so } \underline{\underline{x=\frac{1}{4}}}$	34-b) $\log_4 (\frac{1}{2}) = x$ \Downarrow $4^x = \frac{1}{2}$ $4^x = 2^{-1}$ $4^x = (\sqrt{4})^{-1}$ $4^x = (4^{\frac{1}{2}})^{-1} = 4^{-\frac{1}{2}}$ $\text{so } \underline{\underline{x=-\frac{1}{2}}}$	34-c) $\log_4 8 = x$ \Downarrow $4^x = 8$ $(2^2)^x = 2^3$ $2^{2x} = 2^3$ $2x = 3$ $\underline{\underline{x=\frac{3}{2}}}$
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(4)

$$36-a) \log_3 x = -2$$

\Downarrow

$$3^{-2} = x$$

$$\frac{1}{3^2} = x$$

$$\underline{\underline{\frac{1}{9} = x}}$$

$$36-b) \log_5 125 = x$$

\Downarrow

$$5^x = 125$$

$$5^x = 5^3$$

$$\underline{\underline{x = 3}}$$

$$38-a) \ln x = -1$$

 \Downarrow

$$e^{-1} = x$$

$$\underline{\underline{\frac{1}{e} = x}}$$

$$38-b) \ln\left(\frac{1}{e}\right) = x$$

 \Downarrow

$$e^x = \frac{1}{e}$$

$$e^x = e^{-1}$$

$$\underline{\underline{x = -1}}$$

$$40-a) \log_4 2 = x$$

 \Downarrow

$$4^x = 2$$

$$4^x = \sqrt{4}$$

$$4^x = 4^{\frac{1}{2}}$$

$$\underline{\underline{x = \frac{1}{2}}}$$

$$40-b) \log_4 x = 2$$

 \Downarrow

$$4^2 = x$$

$$\underline{\underline{16 = x}}$$

$$42-a) \log_x 1000 = 3$$

 \Downarrow

$$x^3 = 1000$$

$$x^3 = 10^3$$

$$\underline{\underline{x = 10}}$$

$$42-b) \log_x 25 = 2$$

 \Downarrow

$$x^2 = 25$$

$$x^2 = 5^2$$

$$\underline{\underline{x = 5}}$$

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$$44-a) \log_x 6 = \frac{1}{2}$$

↓

$$\begin{aligned} x^{\frac{1}{2}} &= 6 \\ x^{\frac{1}{2}} &= 36^{\frac{1}{2}} \\ \underline{x = 36} \end{aligned}$$

$$44-b) \log_x 3 = \frac{1}{3}$$

↓

$$\begin{aligned} x^{\frac{1}{3}} &= 3 \\ x^{\frac{1}{3}} &= 27^{\frac{1}{3}} \\ \underline{x = 27} \end{aligned}$$

ex 46 and 48, omit because the course is
ex 45 and 47 without calculator.

$$50) g(x) = \log_4 x = \log(x - (0)) + (0)$$

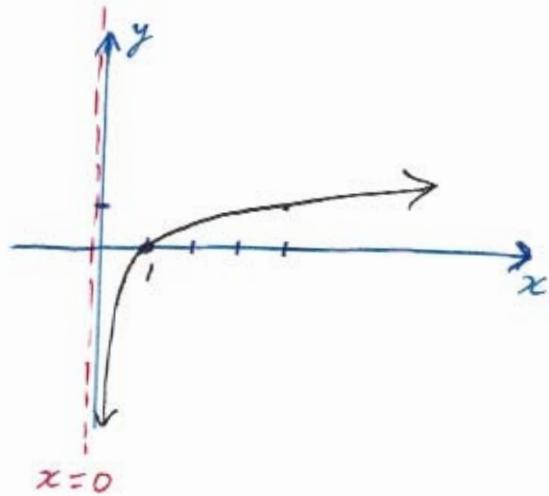
V.A. : $x = 0$

horizontal shift: 0

vertical shift: 0

domain: $x > 0 \Rightarrow 0 < x$
 $(0, \infty)$

range: $(-\infty, \infty)$



$$52) g(x) = 1 + \log x = \log(x - (0)) + (1)$$

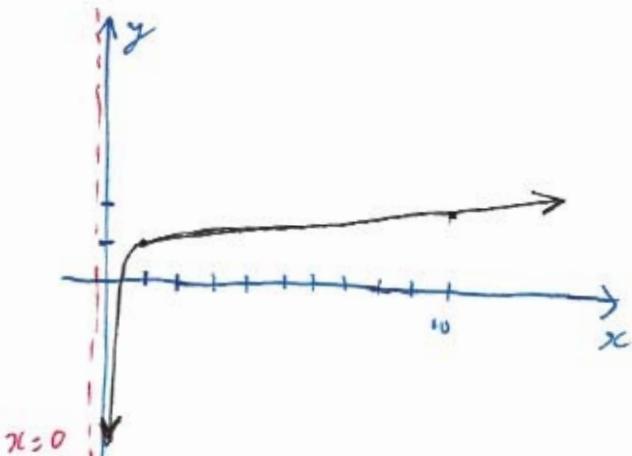
V.A. $x = 0$

horizontal shift: 0

vertical shift: +1

domain: $x > 0 \Rightarrow 0 < x$ $(0, \infty)$

range: $(-\infty, \infty)$



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(6)

54) with point $(\frac{1}{2}, -1)$ and $y = \log_a x$, we get

$$(-1) = \log_a \left(\frac{1}{2}\right) \Rightarrow a^{-1} = \frac{1}{2}$$

$$a^{-1} = 2^{-1}$$

$$a = 2$$

$$\underline{\underline{y = \log_2 x}}$$

56) with point $(9, 2)$ and $y = \log_a x$, we get

$$(2) = \log_a (9) \Rightarrow a^2 = 9$$

$$a^2 = 3^2$$

$$a = 3$$

$$\underline{\underline{y = \log_3 x}}$$

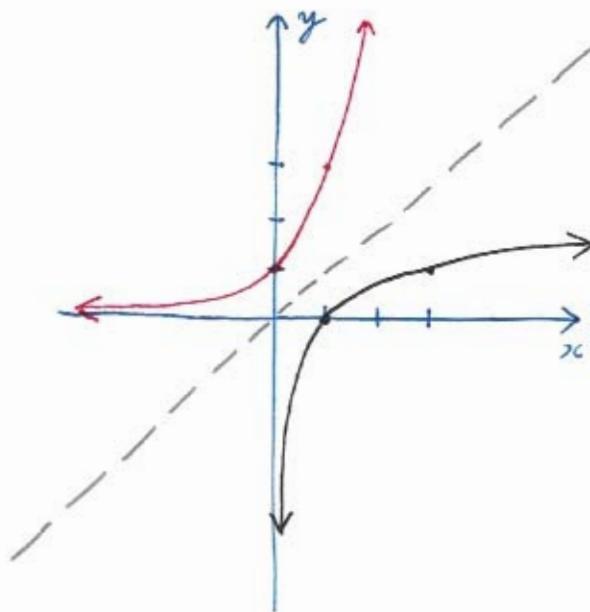
58) $f(x) = \ln(x-2)$

$$\text{V.A. : } x-2=0$$

Graph II

$$x = 2$$

$$60) \quad y = 3^x \quad y = \log_3 x$$



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$$62) f(x) = -\log_{10} x = -\log_{10}(x-(0)) + (0)$$

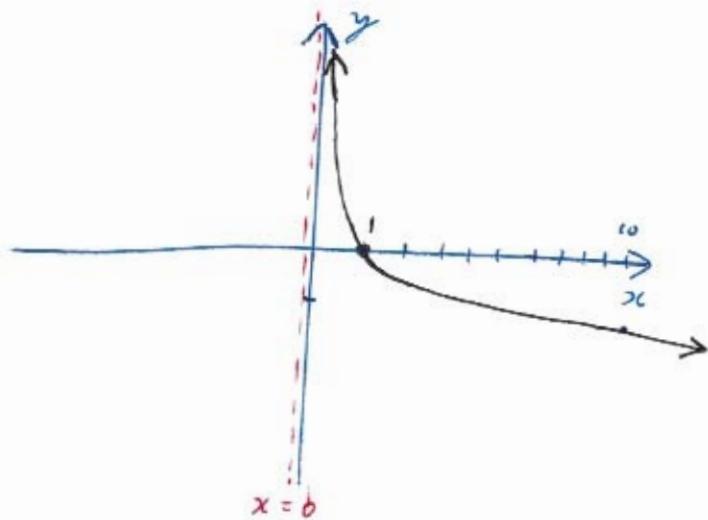
V. A.: $x=0$

horizontal shift: 0

vertical shift: 0

domain: $x > 0$
 $0 < x \in (0, \infty)$ range $(-\infty, \infty)$

"reflect about x-axis"

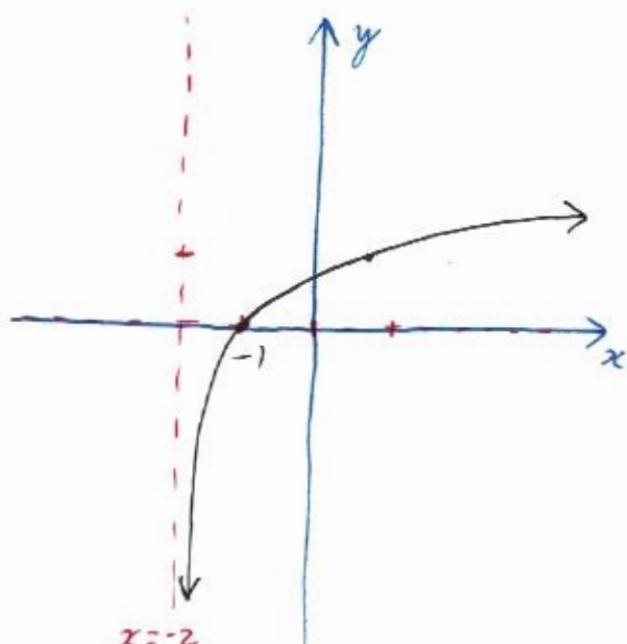


$$64) g(x) = \ln(x+2) = \ln(x-(-2)) + (0)$$

V.A.: $x+2=0$
 $x=-2$

horizontal shift: -2

vertical shift: 0

domain: $x+2 > 0$ $x > -2$ $(-2, \infty)$
 $-2 < x$ range: $(-\infty, \infty)$ overall like $y = \ln x$ 

$$66) g(x) = \log_6 (x-3) = \log_6 (x-(3))+ (0)$$

V.A.: $x-3=0$
 $x=3$

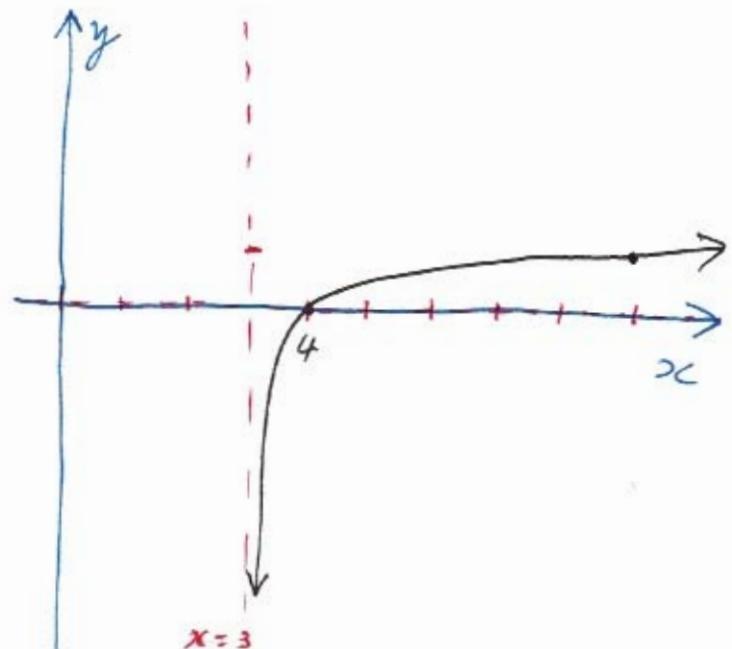
horizontal shift: +3

vertical shift: 0

domain: $x-3 > 0$
 $x > 3 \quad (3, \infty)$
 $3 < x$

range: $(-\infty, \infty)$

overall like $y = \log_6 x$



$$68) y = 1 - \log_{10} x = -\log_{10} (x-(0)) + (1)$$

V.A.: $x=0$

horizontal shift: 0

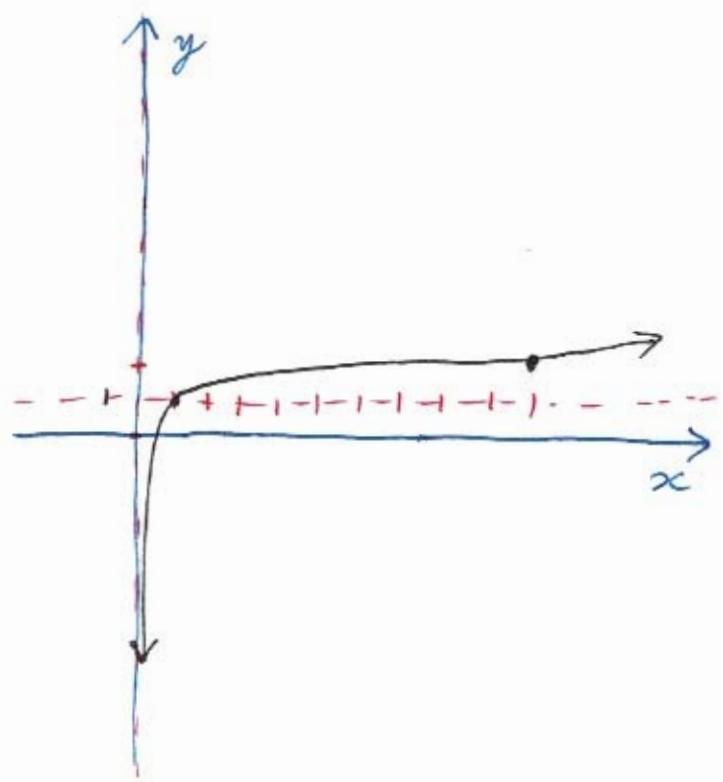
vertical shift: +1

domain: $x > 0 \quad (0, \infty)$
 $0 < x$

range: $(-\infty, \infty)$

overall like $y = -\log_{10} x$

or $y = \log x$



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$$70) y = 1 + \ln(-x) = \ln(-(x-0)) + (1)$$

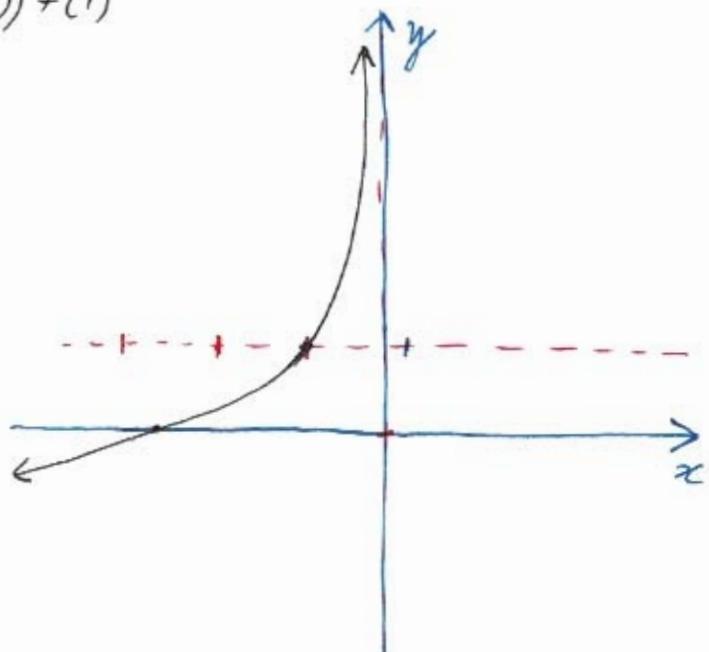
V.A.: $-x = 0$
 $x = 0$

horizontal shift: 0

vertical shift: +1

domain: $-x > 0$

$$\begin{aligned} 0 > x & \quad (-\infty, 0) \\ x < 0 \end{aligned}$$

range: $(-\infty, \infty)$ overall like $y = \ln(-x)$ 

$$72) y = \ln|x| = \ln|x - 0| + (0)$$

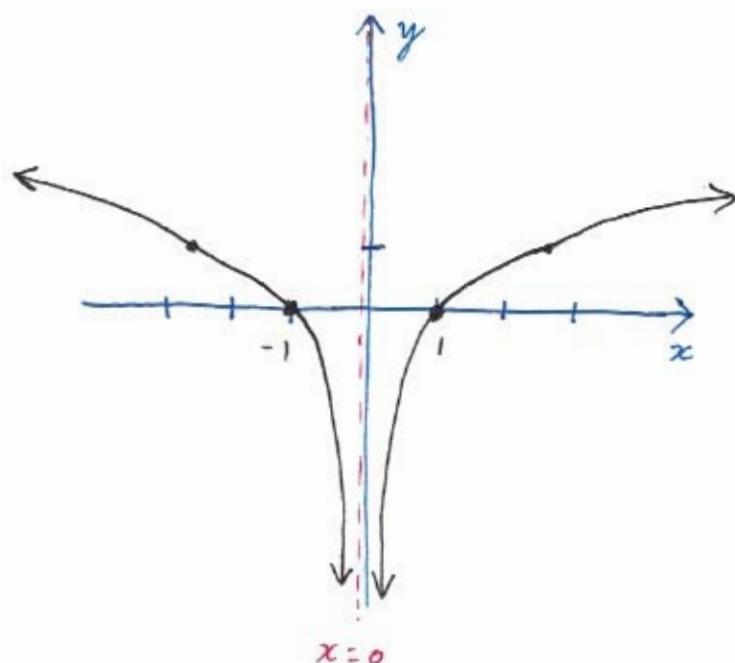
V.A.: $|x| = 0$
 $x = 0$

horizontal shift: 0

vertical shift: 0

domain: $|x| > 0$

$$\begin{array}{ll} -(x) > 0 & +(x) > 0 \\ -x > 0 & x > 0 \\ 0 > x & 0 < x \\ x < 0 & \\ (-\infty, 0) \cup (0, \infty) & \end{array}$$

range: $(-\infty, \infty)$ combination of $y = \ln x$ and $y = \ln(-x)$ 

74) $f(x) = \log_5(8-2x)$

domain: $8-2x > 0$

$$\begin{aligned} 8 &> 2x \\ 4 &> x \\ x &< 4 \end{aligned}$$

$(-\infty, 4)$

76) $g(x) = \ln(x-x^2)$

domain: $x - x^2 > 0$

$$x(1-x) > 0$$

$$\begin{cases} x=0 \\ 1-x=0 \\ x=1 \end{cases}$$

	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
x	neg		pos		pos
$(1-x)$	pos		pos		neg
$x(1-x)$	neg		pos		neg

$(0, 1)$

78) $h(x) = \sqrt{x-2} - \log_5(10-x)$

$$x-2 \geq 0$$

$$10-x > 0$$

$$x \geq 2$$

$$10 > x$$

$$2 \leq x$$

$$x < 10$$

$$[2, \infty)$$

$$(-\infty, 10)$$

domain is the intersection of $[2, \infty)$ and $(-\infty, 10)$

which is $[2, 10)$