2-a) factor of P | 2-b) $P(c)=k$

4) $P(x)=3x^3+9x^2-5x-1$, $D(x)=x+4$

$$
\begin{array}{r}
3x^2-3x+7 \\
x+4\,\overline{\big)\,3x^3+9x^2-5x-1} \\
-(3x^3+12x^2) \\
\hline
-3x^2-5x \\
-(-3x^2-12x) \\
\hline
+7x-1 \\
-(7x+28) \\
\hline
-29
\end{array}
$$

$$\frac{3x^3+9x^2-5x-1}{x+4}=3x^2-3x+7+\frac{(-29)}{x+4}$$

6) $P(x)=6x^3+x^2-12x+5$, $D(x)=3x-4$

$$
\begin{array}{r}
2x^2+3x \\
3x-4\,\overline{\big)\,6x^3+x^2-12x+5} \\
-(6x^3-8x^2) \\
\hline
+9x^2-12x \\
-(9x^2-12x) \\
\hline
0x+5
\end{array}
$$

$$\frac{6x^3+x^2-12x+5}{3x-4}=2x^2+3x+\frac{(+5)}{3x-4}$$

8) $P(x) = 2x^5 + x^3 - 2x^2 + 3x - 5$, $\quad D(x) = x^2 - 3x + 1$

$$
\begin{array}{r|l}
 & 2x^3 + 6x^2 + 16x + 40 \\
\hline
x^2 - 3x + 1 & 2x^5 + 0x^4 + x^3 - 2x^2 + 3x - 5 \\
 & -(2x^5 - 6x^4 + 2x^3) \\
 & \quad +6x^4 - 2x^3 - 2x^2 \\
 & \quad -(6x^4 - 18x^3 + 6x^2) \\
 & \qquad +16x^3 - 8x^2 + 3x \\
 & \qquad -(16x^3 - 48x^2 + 16x) \\
 & \qquad\quad +40x^2 - 13x - 5 \\
 & \qquad\quad -(40x^2 - 120x + 40) \\
 & \qquad\qquad +107x - 45
\end{array}
$$

$$\frac{2x^5 + x^3 - 2x^2 + 3x - 5}{x^2 - 3x + 1}$$

$$= 2x^3 + 6x^2 + 16x + 40 + \frac{(+107x - 45)}{x^2 - 3x + 1}$$

---

10) $P(x) = x^4 + 2x^3 - 10x$, $\quad D(x) = x - 3$

$$
\begin{array}{r|l}
 & x^3 + 5x^2 + 15x + 35 \\
\hline
x - 3 & x^4 + 2x^3 + 0x^2 - 10x + 0 \\
 & -(x^4 - 3x^3) \\
 & \quad +5x^3 + 0x^2 \\
 & \quad -(5x^3 - 15x^2) \\
 & \qquad +15x^2 - 10x \\
 & \qquad -(15x^2 - 45x) \\
 & \qquad\quad +35x + 0 \\
 & \qquad\quad -(35x - 105) \\
 & \qquad\qquad +105
\end{array}
$$

$$x^4 + 2x^3 - 10x$$

$$= (x-3)(x^3 + 5x^2 + 15x + 35) + (+105)$$

12) $P(x) = 4x^3 + 7x + 9$, $D(x) = 2x + 1$

$$
\begin{array}{r|l}
 & 2x^2 - x + 3 \\
\hline
2x+1 & 4x^3 + 0x^2 + 7x + 9 \\
 & -(4x^3 + 2x^2) \\
 & \quad -2x^2 + 7x \\
 & \quad -(-2x^2 - x) \\
 & \qquad +6x + 9 \\
 & \qquad -(6x + 3) \\
 & \qquad\quad +6
\end{array}
$$

$$4x^3 + 7x + 9 = (2x+1)(2x^2 - x + 3) + (+6)$$

---

14) $P(x) = 27x^5 - 9x^4 + 3x^2 - 3$, $D(x) = 3x^2 - 3x + 1$

$$
\begin{array}{r|l}
 & 9x^3 + 6x^2 + 3x + 2 \\
\hline
3x^2-3x+1 & 27x^5 - 9x^4 + 0x^3 + 3x^2 + 0x - 3 \\
 & -(27x^5 - 27x^4 + 9x^3) \\
 & \quad +18x^4 - 9x^3 + 3x^2 \\
 & \quad -(18x^4 - 18x^3 + 6x^2) \\
 & \qquad +9x^3 - 3x^2 + 0x \\
 & \qquad -(9x^3 - 9x^2 + 3x) \\
 & \qquad\quad +6x^2 - 3x - 3 \\
 & \qquad\quad -(6x^2 - 6x + 2) \\
 & \qquad\qquad +3x - 5
\end{array}
$$

$$27x^5 - 9x^4 + 3x^2 - 3$$

$$= (3x^2 - 3x + 1)(9x^3 + 6x^2 + 3x + 2) + (+3x - 5)$$

16) $\dfrac{x^3+2x^2-x+1}{x+3}$

$Q(x) = x^2 - x + 2$

$R(x) = -5$

$$
\begin{array}{r}
x^2 - x + 2 \\
x+3 \overline{\big) x^3 + 2x^2 - x + 1} \\
-(x^3 + 3x^2) \\
\hline
-x^2 - x \\
(-x^2 - 3x) \\
\hline
+2x + 1 \\
-(2x+6) \\
\hline
-5
\end{array}
$$

$$\frac{x^3+2x^2-x+1}{x+3} = x^2 - x + 2 + \frac{(-5)}{x+3}$$

---

18) $\dfrac{x^3+3x^2+4x+3}{3x+6}$

$Q(x) = \frac{1}{3}x^2 + \frac{1}{3}x + \frac{2}{3}$

$R(x) = -1$

$$
\begin{array}{r}
\frac{1}{3}x^2 + \frac{1}{3}x + \frac{2}{3} \\
3x+6 \overline{\big) x^3 + 3x^2 + 4x + 3} \\
-(x^3 + 2x^2) \\
\hline
x^2 + 4x \\
-(x^2 + 2x) \\
\hline
+2x + 3 \\
-(2x+4) \\
\hline
-1
\end{array}
$$

$$\frac{x^3+3x^2+4x+3}{3x+6} = \frac{1}{3}x^2 + \frac{1}{3}x + \frac{2}{3} + \frac{(-1)}{3x+6}$$

20) $\dfrac{x^4 - 3x^3 + x - 2}{x^2 - 5x + 1}$

$Q(x) = x^2 + 2x + 9$

$R(x) = 44x - 11$

$$
\begin{array}{r|l}
 & x^2 + 2x + 9 \\
\hline
x^2 - 5x + 1 & x^4 - 3x^3 + 0x^2 + x - 2 \\
 & -(x^4 - 5x^3 + x^2) \\
\hline
 & +2x^3 - x^2 + x \\
 & -(2x^3 - 10x^2 + 2x) \\
\hline
 & +9x^2 - x - 2 \\
 & -(9x^2 - 45x + 9) \\
\hline
 & +44x - 11
\end{array}
$$

$$\frac{x^4 - 3x^3 + x - 2}{x^2 - 5x + 1} = x^2 + 2x + 9 + \frac{(+44x - 11)}{x^2 - 5x + 1}$$

---

22) $\dfrac{9x^2 - x + 5}{3x^2 - 7x}$

$Q(x) = 3$

$R(x) = 20x - 5$

$$
\begin{array}{r|l}
 & 3 \\
\hline
3x^2 - 7x + 0 & 9x^2 - x + 5 \\
 & -(9x^2 - 21x + 0) \\
\hline
 & +20x - 5
\end{array}
$$

$$\frac{9x^2 - x + 5}{3x^2 - 7x} = 3 + \frac{(+20x - 5)}{3x^2 - 7x}$$

24) $\dfrac{2x^5-7x^4-13}{4x^2-6x+8}$

$$
\begin{array}{r|l}
 & \frac{1}{2}x^3 - x^2 - \frac{5}{2}x - \frac{7}{4} \\
\hline
4x^2-6x+8 & 2x^5-7x^4+0x^3+0x^2+0x-13 \\
 & -(2x^3-3x^4+4x^3) \\
 & -4x^4-4x^3+0x^2 \\
 & -(-4x^4+6x^3-8x^2) \\
 & -10x^3+8x^2+0x \\
 & -(-10x^3+15x^2-20x) \\
 & -7x^2+20x-13 \\
 & -(-7x^2+\frac{21}{2}x-14) \\
 & +\frac{19}{2}x+1
\end{array}
$$

$Q(x) = \frac{1}{2}x^3 - x^2 - \frac{5}{2}x - \frac{7}{4}$

$R(x) = \frac{19}{2}x + 1$

$$\frac{2x^5-7x^4-13}{4x^2-6x+8} = \frac{1}{2}x^3 - x^2 - \frac{5}{2}x - \frac{7}{4} + \frac{\left(\frac{+19}{2}x+1\right)}{4x^2-6x+8}$$