19500 section 3.2

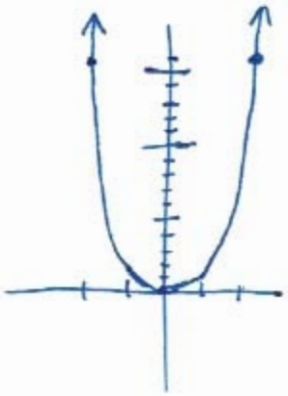
2-a) $y \to +\infty$ as $x \to \infty$; $y \to -\infty$ as $x \to -\infty$

2-b) $y \to +\infty$ as $x \to \infty$; $y \to +\infty$ as $x \to -\infty$

4-a) is only one could not possibly be true about the polynomial function.

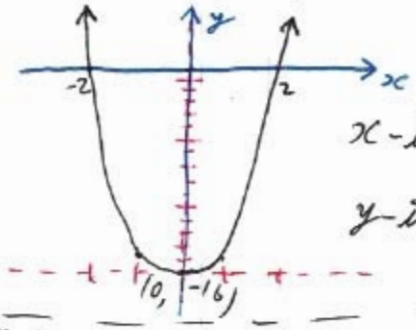
6) $y = x^4$

x	y
-2	16
-1	1
0	0
1	1
2	16



a) $P(x) = x^4 - 16 = (x-(0))^4 + (-16)$

"vertex": $(0, -16)$



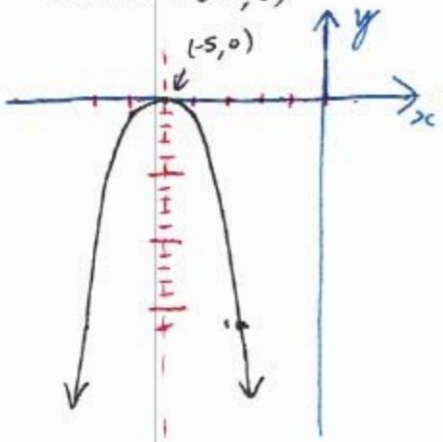
x-int: $x = -2, x = 2$

y-int: $y = -16$

b) $P(x) = -(x+5)^4$

$= -(x-(-5))^4 + (0)$

"vertex": $(-5, 0)$



x-int: $x = -5$

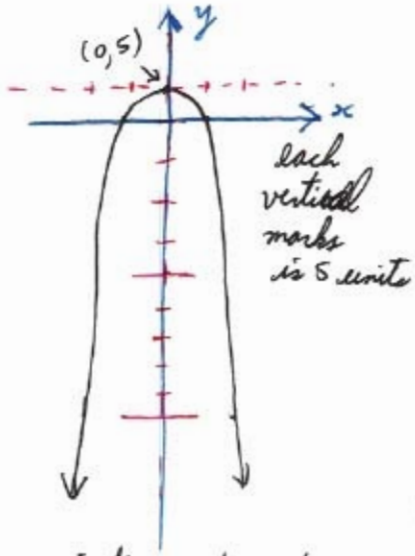
y-int: set $x = 0$ and solve for y

$y = -((0)+5)^4 = \underline{\underline{-625}}$

c) $P(x) = -5x^4 + 5$

$= -5(x-(0))^4 + (5)$

"vertex": $(0, 5)$



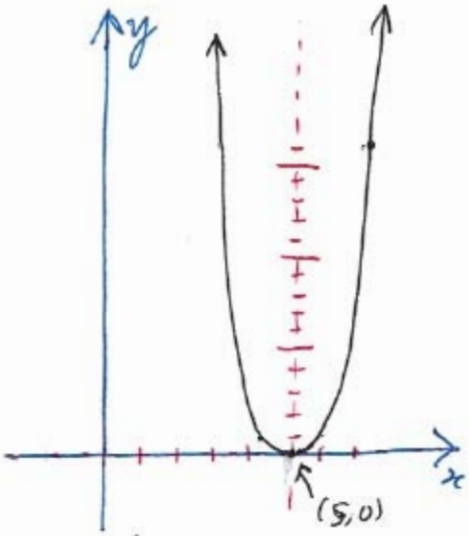
x-int: $x = -1, x = 1$

y-int: $y = 5$

d) $P(x) = (x-5)^4$

$= (x-(5))^4 + (0)$

"vertex": $(5, 0)$

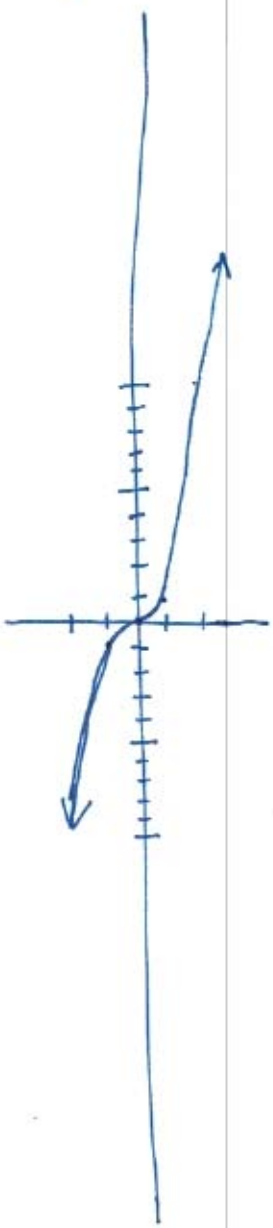


x-int: $x = 5$

y-int: $y = ((0)-5)^4 = \underline{\underline{625}}$

8) $y = x^5$

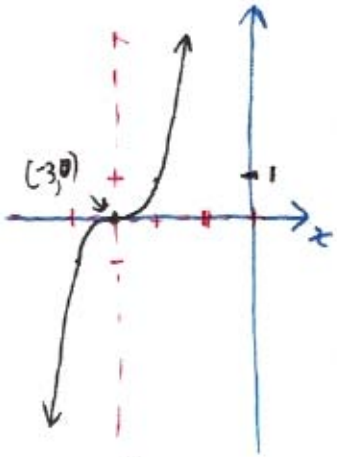
x	y
-2	-32
-1	1
0	0
1	1
2	32



a) $P(x) = (x+3)^5$

$= (x-(-3))^5 + (0)$

"center": $(-3, 0)$



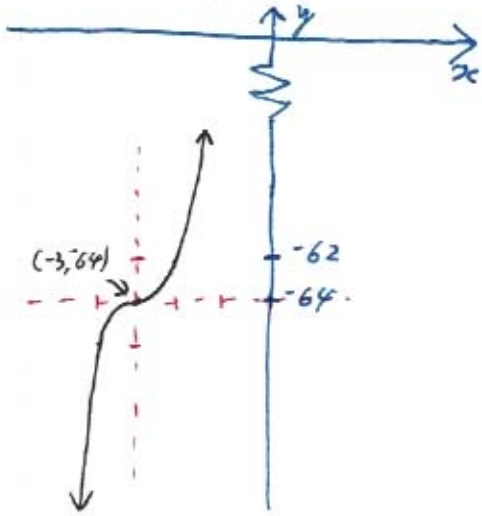
x-int: $x = -3$

y-int: $y = ((0)+3)^5 = 243$

b) $Q(x) = 2(x+3)^5 - 64$

$= 2(x-(-3))^5 + (-64)$

"center": $(-3, -64)$



y-int: $y = 2((0)+3)^5 - 64 = 422$

x-int: set $y=0$ and solve for x

$0 = 2(x+3)^5 - 64$

$64 = 2(x+3)^5$

$32 = (x+3)^5$

$\sqrt[5]{32} = (x+3)$

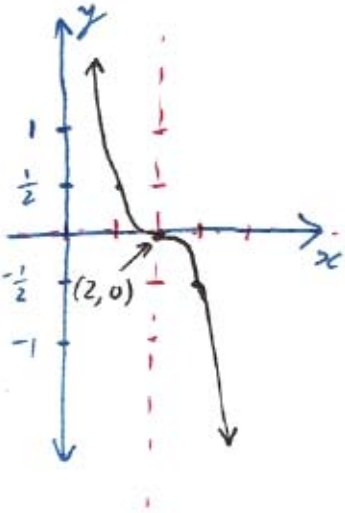
$2 = x+3$

$\underline{\underline{-1 = x}}$

c) $R(x) = \frac{-1}{2}(x-2)^5$

$= \frac{-1}{2}(x-(2))^5 + (0)$

"center": $(2, 0)$



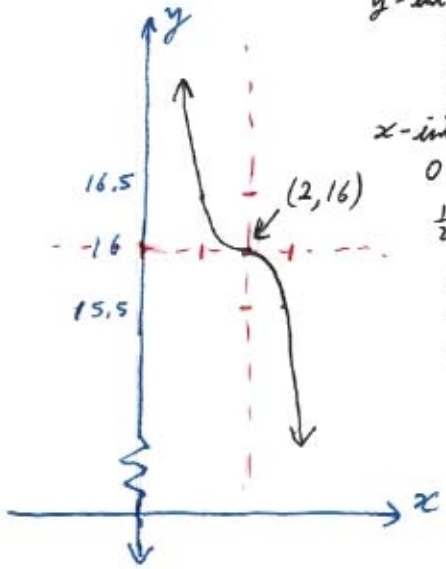
y-int: $y = \frac{-1}{2}((0)-2)^5 = 16$

x-int: $x = 2$

d) $S(x) = \frac{-1}{2}(x-2)^5 + 16$

$= \frac{-1}{2}(x-(2))^5 + (16)$

"center": $(2, 16)$



y-int: $y = \frac{-1}{2}((0)-2)^5 + 16$

$y = 24$

x-int:

$0 = \frac{-1}{2}(x-2)^5 + 16$

$\frac{1}{2}(x-2)^5 = 16$

$(x-2)^5 = 32$

$x - 2 = \sqrt[5]{32}$

$x - 2 = 2$

$\underline{\underline{x = 4}}$

19500 section 3.2

10) $Q(x) = -x^2(x^2-4)$

y-int: $y = -(0)^2((0)^2-4) = 0$

x-int: $0 = -x^2(x^2-4)$

$0 = -(x+2)^1(x)^2(x-2)^1$

$x+2=0$	$(x)^2=0$	$x-2=0$
$x=-2$	$x=0_T$	$x=2$

end behavior like $y=-x^4$
both ends going down with
x-int crossing at $x=-2$ and $x=2$,
touching at $x=0_T$. Graph I

---

12) $S(x) = \frac{1}{2}x^6 - 2x^4$

end behavior like $y=x^6$
both ends going up with
x-int crossing at $x=-2$ and $x=2$,
touching at $x=0_T$.

y-int: $y = \frac{1}{2}(0)^6 - 2(0)^4 = 0$

x-int: $0 = \frac{1}{2}x^6 - 2x^4$

$0 = \frac{1}{2}x^4(x^2-4)$

$0 = \frac{1}{2}(x+2)^1(x)^4(x-2)^1$

$x+2=0$	$(x)^4=0$	$x-2=0$
$x=-2$	$x=0_T$	$x=2$

Graph II

---

14) $U(x) = -x^3 + 2x^2$

end behavior like $y=-x^3$
overall decreasing graph.
x-int touching at $x=0_T$
and crossing at $x=2$

y-int: $y = -(0)^3 + 2(0)^2 = 0$

x-int: $0 = -x^3 + 2x^2$

$0 = -(x)^2(x-2)$

$(x)^2=0$	$x-2=0$
$x=0_T$	$x=2$

Graph IV

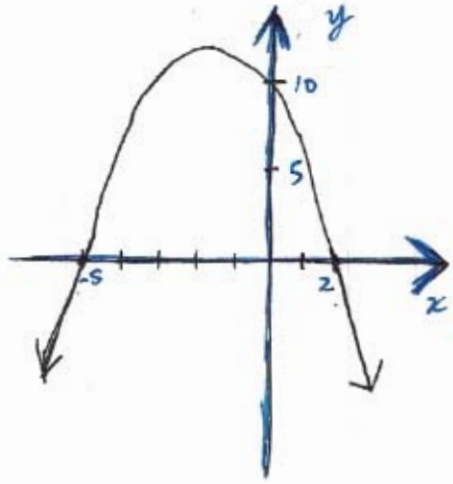
19500 section 3.2

16) $P(x) = (2-x)(x+5)$

y-int: $y = (2-(0))((0)+5) = 10$

x-int: $0 = (2-x)^1(x+5)^1$

$x+5=0$	$2-x=0$
$x=-5$	$x=2$



overall like $y=-x^2$

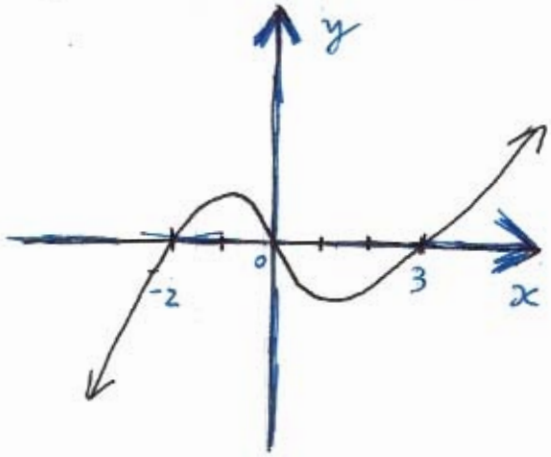
---

18) $P(x) = x(x-3)(x+2)$

y-int: $y = (0)((0)-3)((0)+2) = 0$

x-int: $0 = (x)^1(x-3)^1(x+2)^1$

$x+2=0$	$x=0$	$x-3=0$
$x=-2$		$x=3$



overall like $y=x^3$

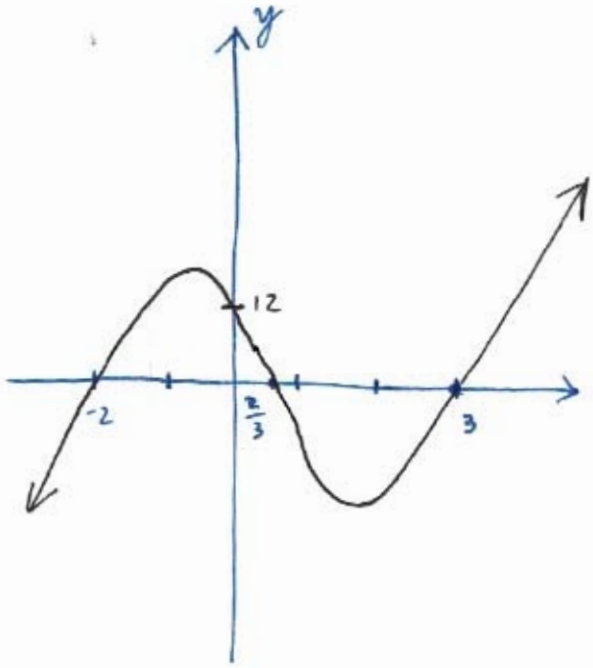
---

20) $P(x) = (x-3)(x+2)(3x-2)$

y-int: $y = ((0)-3)((0)+2)(3(0)-2) = 12$

x-int: $0 = (x-3)^1(x+2)^1(3x-2)^1$

$x+2=0$	$3x-2=0$	$x-3=0$
$x=-2$	$3x=2$	$x=3$
	$x=\frac{2}{3}$	



overall like $y=3x^3$

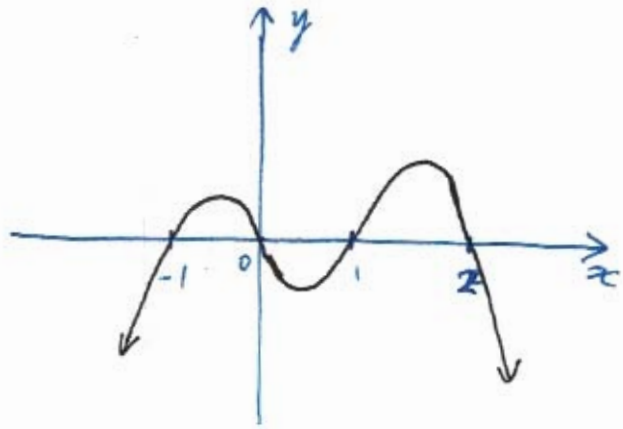
19500 section 3.2

22) $P(x) = x(x+1)(x-1)(2-x)$

y-int: $y = (0)((0)+1)((0)-1)(2-(0)) = 0$

x-int: $0 = (x)^1(x+1)^1(x-1)^1(2-x)^1$

$x+1=0$	$x=0$	$x-1=0$	$2-x=0$
$x=-1$		$x=1$	$x=2$



overall like $y=-x^4$

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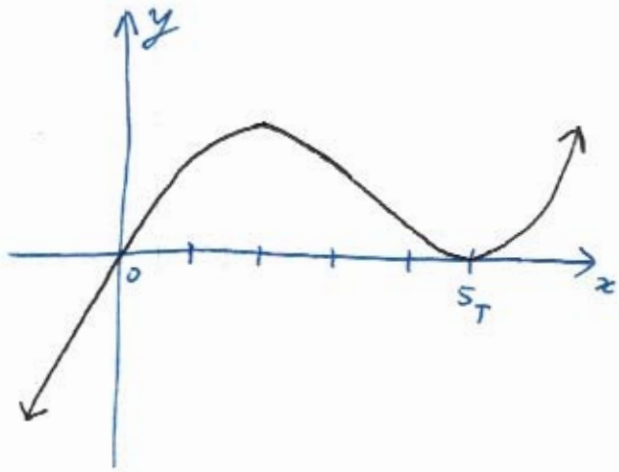
24) $P(x) = \frac{1}{5}x(x-5)^2$

y-int: $y = \frac{1}{5}(0)((0)-5)^2 = 0$

x-int: $0 = \frac{1}{5}(x)^1(x-5)^2$

$x=0$	$(x-5)^2=0$
	$x-5=0$
	$x=5_T$



overall like $y=x^3$

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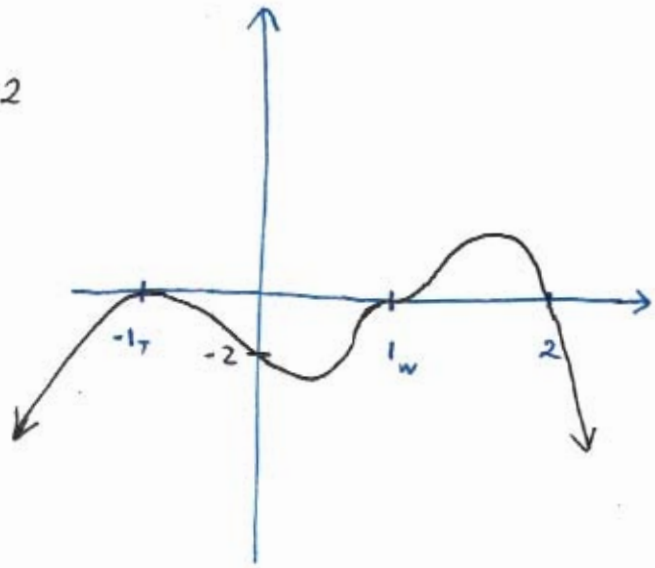
26) $P(x) = -(x+1)^2(x-1)^3(x-2)$

y-int: $y = -((0)+1)^2((0)-1)^3((0)-2) = -2$

x-int: $0 = -(x+1)^2(x-1)^3(x-2)^1$

$(x+1)^2=0$	$(x-1)^3=0$	$x-2=0$
$x+1=0$	$x-1=0$	$x=2$
$x=-1_T$	$x=1_W$	



overall like $y=-x^6$

19500 section 3.2

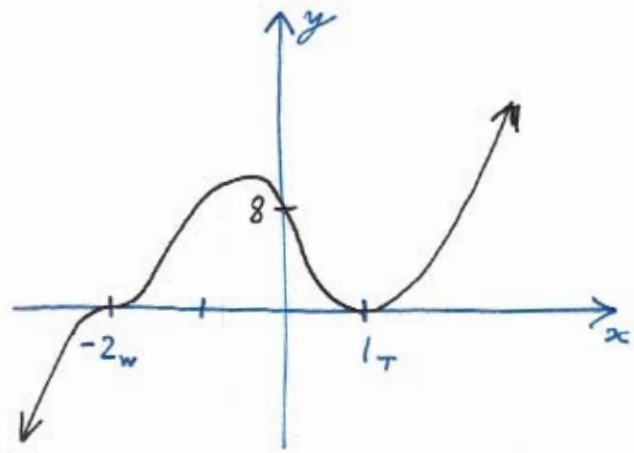
28) $P(x) = (x-1)^2(x+2)^3$

y-int: $y = ((0)-1)^2((0)+2)^3 = 8$

x-int: $0 = (x-1)^2(x+2)^3$

$(x+2)^3 = 0$	$(x-1)^2 = 0$
$x+2 = 0$	$x-1 = 0$
$x = -2_W$	$x = 1_T$

overall like $y = x^5$



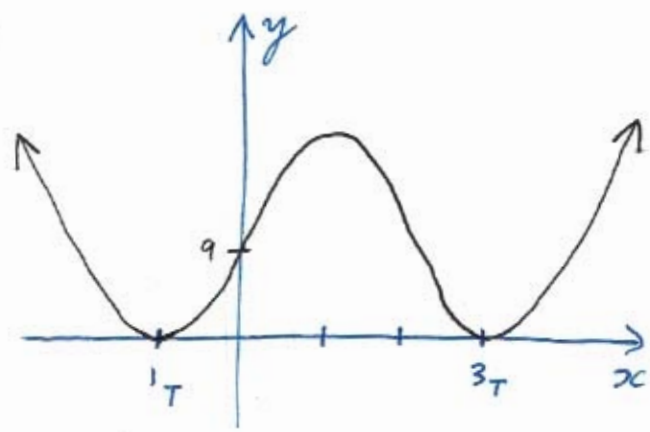
30) $P(x) = (x-3)^2(x+1)^2$

y-int: $y = ((0)-3)^2((0)+1)^2 = 9$

x-int: $0 = (x-3)^2(x+1)^2$

$(x+1)^2 = 0$	$(x-3)^2 = 0$
$x+1 = 0$	$x-3 = 0$
$x = -1_T$	$x = 3_T$

overall like $y = x^4$



32) $P(x) = x^3 + 2x^2 - 8x$

y-int: $y = (0)^3 + 2(0)^2 - 8(0) = 0$

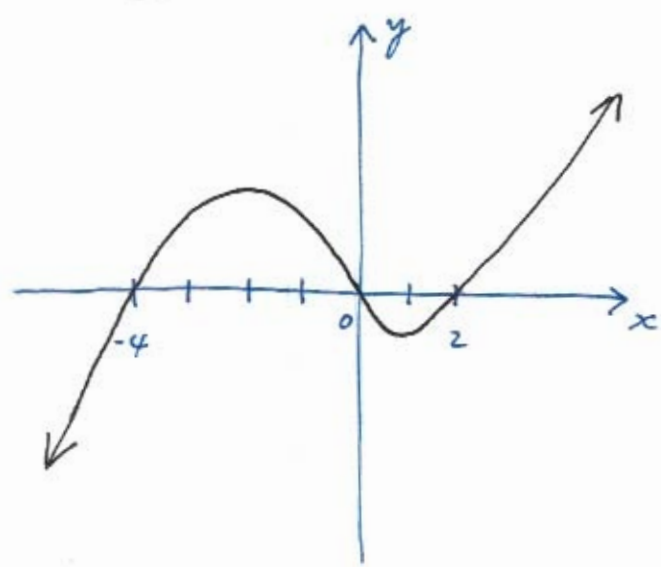
x-int: $0 = x^3 + 2x^2 - 8x$

$0 = x(x^2 + 2x - 4)$

$0 = (x+4)^1(x)^1(x-2)^1$

$x+4 = 0$	$x = 0$	$x-2 = 0$
$x = -4$		$x = 2$

overall like $y = x^3$



19500 section 3.2

34) $P(x) = -2x^3 - x^2 + x$

y-int: $y = -2(0)^3 - (0)^2 + (0) = 0$

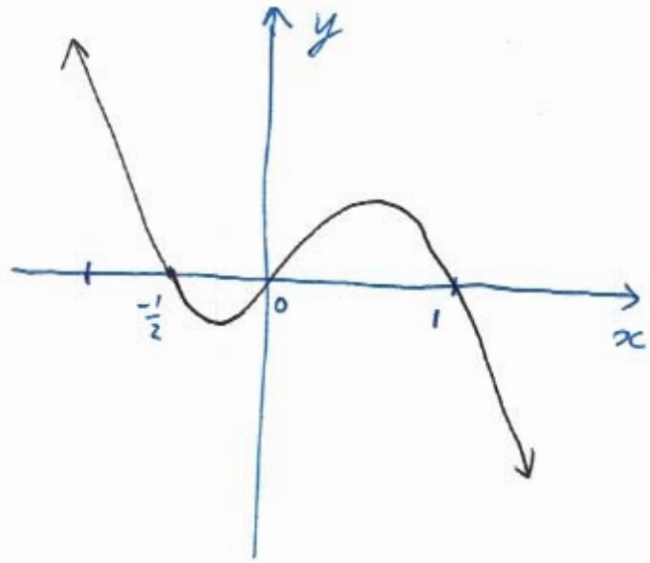
x-int: $0 = -2x^3 - x^2 + x$

$0 = -x(2x^2 - x + 1)$

$0 = -(2x+1)^1(x)^1(x-1)^1$

$2x+1=0$	$x=0$	$x-1=0$
$2x=-1$		$x=1$
$x=\frac{-1}{2}$		

overall like $y = -2x^3$



36) $P(x) = x^5 - 9x^3$

y-int: $y = (0)^5 - 9(0)^3 = 0$

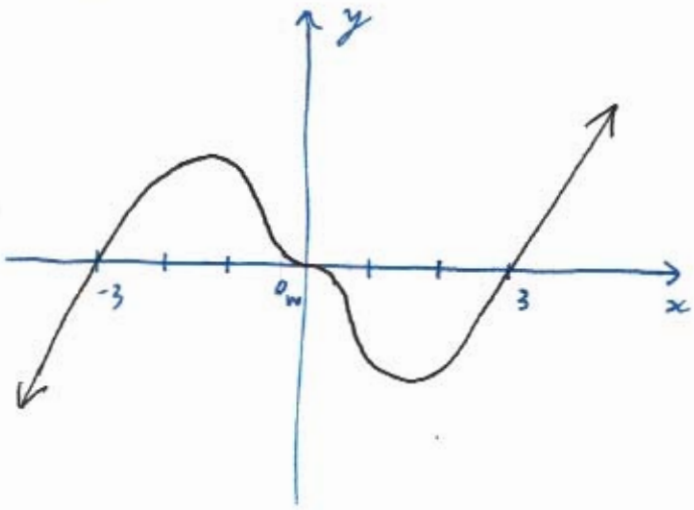
x-int: $0 = x^5 - 9x^3$

$0 = x^3(x^2 - 9)$

$0 = (x+3)^1(x)^3(x-3)^1$

$x+3=0$	$(x)^3=0$	$x-3=0$
$x=-3$	$x=0_w$	$x=3$

overall like $y = x^5$



38) $P(x) = x^3 + 3x^2 - 4x - 12$

y-int: $y = (0)^3 + 3(0)^2 - 4(0) - 12 = -12$

x-int: $0 = x^3 + 3x^2 - 4x - 12$

$0 = (x^3 + 3x^2) - (4x + 12)$

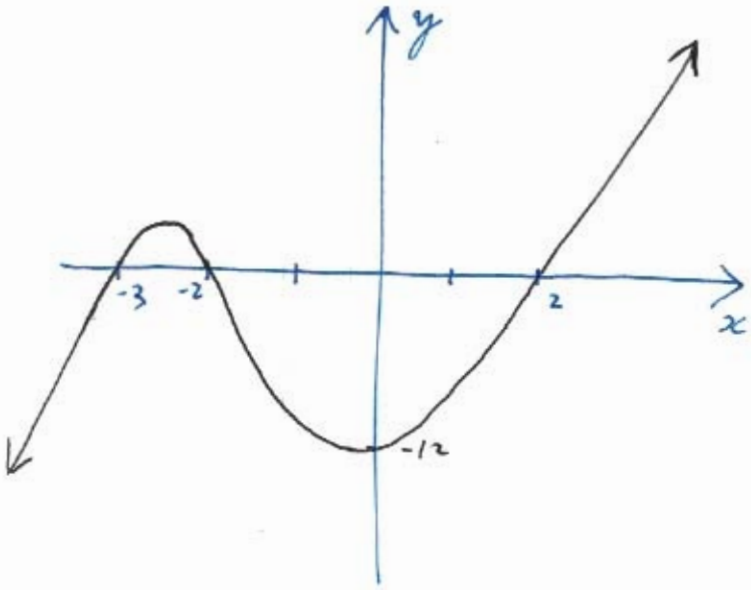
$0 = x^2(x+3) - 4(x+3)$

$0 = (x+3)(x^2 - 4)$

$0 = (x+3)^1(x+2)^1(x-2)^1$

$x+3=0$	$x+2=0$	$x-2=0$
$x=-3$	$x=-2$	$x=2$

overall like $y = x^3$



19500 section 3.2

40) $P(x) = \frac{1}{8}(2x^4+3x^3-16x-24)^2$

overall like $y = \frac{4}{8}x^8 = \frac{1}{2}x^8$

y-int: $y = \frac{1}{8}(2(0)^4+3(0)^3-16(0)-24)^2$

$= \frac{1}{8}(-24)^2 = 72$

x-int: $0 = \frac{1}{8}(2x^4+3x^3-16x-24)^2$

$0 = \frac{1}{8}((2x^4+3x^3)-(16x+24))^2$

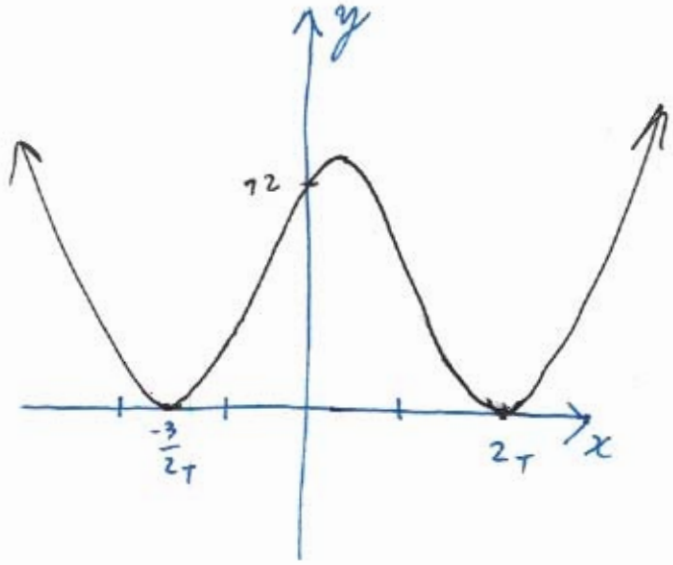
$0 = \frac{1}{8}(x^3(2x+3)-8(2x+3))^2$

$0 = \frac{1}{8}((2x+3)(x^3-8))^2$

$0 = \frac{1}{8}((2x+3)(x-2)(x^2+2x+4))^2$

$0 = \frac{1}{8}(2x+3)^2(x-2)^2(x^2+2x+4)^2$

$(2x+3)^2=0$	$(x-2)^2=0$	$(x^2+2x+4)^2=0$
$2x+3=0$	$x-2=0$	$x^2+2x+4=0$
$x=\frac{-3}{2}$ T	$x=2$ T	discard, not real number solution



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42) $P(x) = x^4-2x^3+8x-16$

y-int: $y = (0)^4-2(0)^3+8(0)-16 = -16$

x-int: $0 = x^4-2x^3+8x-16$

$0 = (x^4-2x^3)+(8x-16)$

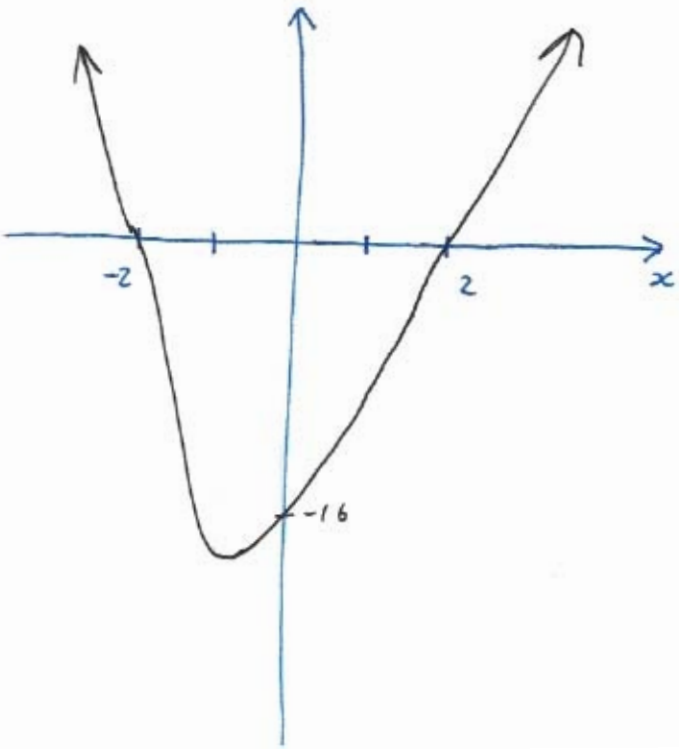
$0 = x^3(x-2)+8(x-2)$

$0 = (x-2)(x^3+8)$

$0 = (x-2)^1(x+2)^1(x^2-2x+4)^1$

$x+2=0$	$x-2=0$	$x^2-2x+4=0$
$x=-2$	$x=2$	discard, not real number solution

overall like $y=x^4$



44) $P(x) = x^6 - 2x^3 + 1$

y-int: $y = (0)^6 - 2(0)^3 + 1 = 1$

x-int: $0 = x^6 - 2x^3 + 1$

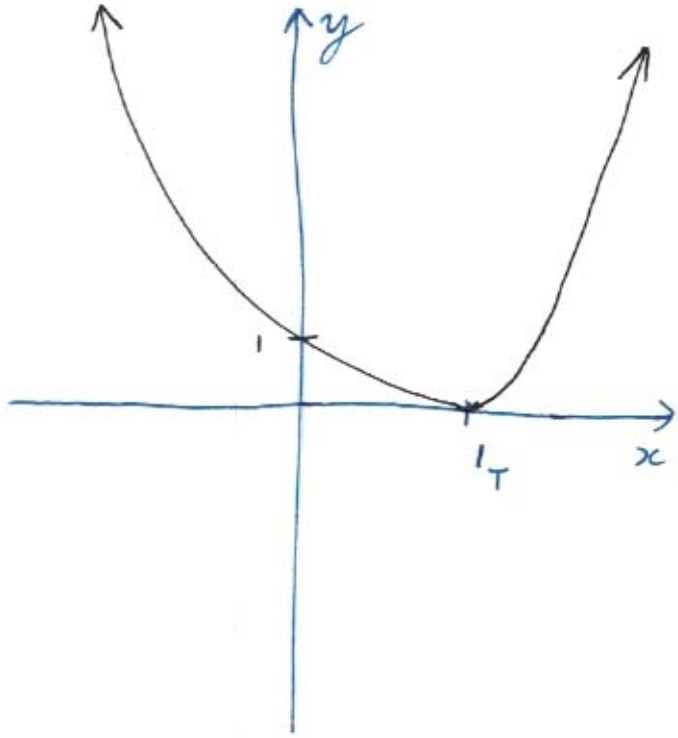
$0 = (x^3 - 1)(x^3 - 1)$

$0 = (x^3 - 1)^2$

$0 = ((x-1)(x^2+x+1))^2$

$0 = (x-1)^2(x^2+x+1)^2$

$(x-1)^2 = 0$	$(x^2+x+1)^2 = 0$
$x - 1 = 0$	$x^2 + x + 1 = 0$
$x = 1_T$	discard, not real number solution



overall like $y = x^6$

52) $P(x) = \frac{2}{9}x^3 - x^2$

a) y-int: $y = \frac{2}{9}(0)^3 - (0)^2 = 0$

x-int: $0 = \frac{2}{9}x^3 - x^2$

$0 = \frac{1}{9}x^2(2x-1)$

$0 = \frac{1}{9}(x)^2(2x-1)^1$

$(x)^2 = 0$	$2x - 1 = 0$
$x = 0_T$	$2x = 1$
	$x = \frac{1}{2}$

b) local max: $f(0) = 0$

local min: $f(3) = -3$

54) $P(x) = \frac{1}{9}x^4 - \frac{4}{9}x^3$

a) y-int: $y = \frac{1}{9}(0)^4 - \frac{4}{9}(0)^3 = 0$

x-int: $0 = \frac{1}{9}x^4 - \frac{4}{9}x^3$

$0 = \frac{1}{9}x^3(x-4)$

$0 = \frac{1}{9}(x)^3(x-4)^1$

$(x)^3 = 0$	$x - 4 = 0$
$x = 0_W$	$x = 4$

b) local max: none

local min: $f(3) = -3$