MATHE 7700E MODERN ALGEBRA
SPRING 2012

MATTHEW AUTH, LECTURER OF MATHEMATICS

General Information

Course information like homework assignments, exams dates, class topics, and additional materials will be posted on our course website: `http://math.sci.ccny.cuny.edu/people?name=Matthew_Auth`. Office hours will be held in NAC 6-288 Modays 8:45–9:15 and Wednesdays 7–7:30 or by appointment.

Course Goals

To understand how symmetry manifests itself in the high school math curriculum. Our students have an intuitive sense of geometric symmetries like mirror symmetry or rotational symmetry. They encounter geometric symmetries during high school while studying Euclidean geometry and while graphing functions. However it is not immediately apparent that there are symmetries at work in algebra when solving quadratic equations $ax^2 + bx + c = 0$. In this course we will find that there are symmetries involved in solving general nonlinear polynomial equations $p(x) = 0$.

To see this will require a substantial effort. The story of the search for the roots of the cubic equation $y^3 + hy + k = 0$ is one of the most intriguing in the history of mathematics. Many bright minds and colorful characters in Renaissance Italy set out to find the roots. In the end they were given by the formulas

$$\sqrt[3]{\frac{1}{2}(-k + \sqrt{k^2 + 4h^3})} + \sqrt[3]{\frac{1}{2}(-k - \sqrt{k^2 + 4h^3})}.$$

The solutions of the quartic $ax^4 + bx^3 + cx^2 + dx + e = 0$ were found soon thereafter and the formulas appear correspondingly more complicated. The general quintic, $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$, however is unsolvable.

This is the great mystery of the course. How can there be formulas like the quadratic formula to solve the general degree three and degree four polynomials but no such formula for degree five—or for any higher degree for that matter? Something must break down between degrees four and five.

It took the genius of the mathematicians Lagrange, Abel, and Galois to discover the reason: It is symmetry. We will find in the course that to each equation we can associate a mathematical structure called a group. Whenever there is symmetry, mathematicians introduce groups. We will study groups of symmetries of geometric shapes as well as the groups associated to polynomial equations.

In fact this viewpoint can be used to unify seemingly disparate ideas in middle school and high school mathematics. For instance, by the end of the course you will

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understand how the symmetries of a equilateral triangle are related to the formula to solve a general cubic equation

$$ax^3 + bx^2 + cx + d = 0.$$

Required Texts

(1) "Theory of Algebraic Equations" by L. E. Dickson (free pdf copy on our course website).
(2) "Abel's Proof" MIT Press 2004 by Peter Pesic
(3) "Symmetry" Princeton University Press, 1983, Hermann Weyl

Grading

I grade the homework assignments but I do not grade all the problems. I grade a small randomly chosen subset of the assigned problems. Before computing your final course grade, I will drop your lowest homework score.

Final Grades will be determined using the following averaging:

Midterm1	25%
Midterm2	25%
Final Exam	25%
Homework	10%
Attendance and participation	15%

Course Meeting Times

During class I will try to give you some time to work in small groups on the homework assignments. Therefore, in order to get the most out of class, it important to keep up with the assigned readings and problems. Even if you do not have enough time to do every assigned problem, try to at least read the problems before class.

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