Homework 3 Due Wednesday 7 March

1. Read pages 6–14 in Dickson's text.
2. In section 6, suppose that you have found a solution $y_1 = x_1x_2 + x_3x_4$ to the resolvent cubic $(y - y_1)(y - y_2)(y - y_3) = 0$ where x_1, x_2, x_3, x_4 are solutions to the given quartic, $x^4 + ax^3 + bx^2 + cx + d$ (remember at this stage you are supposing that you know y_1 and, obviously, you know the coefficients a, b, c, and d of the original equation—you do not necessarily know x_1, x_2, x_3, x_4). Show that x_1x_2 and x_3x_4 are the two solutions to the quadratic $z^2 - y_1z + d = 0$.
3. Following the previous problem. Explain how to use x_1x_2 and x_3x_4 and the coefficients, a, b, c, d, to then find x_1+x_2 and x_3+x_4. Finally use what you have found to write two quadratic equations: one having solutions x_1, x_2 and the other having solutions x_3, x_4.
4. Find an integer n so that $s^n = I$ for every element of the symmetric group on 3 letters.
5. Write the substitution $s = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}$ as disjoint cycles = circular substitutions affecting different letters.
6. Write the substitution $s = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ as disjoint cycles.
7. Express the substitution (13)(132)(13) as the product of disjoint cycles.
8. Compute $s^{-1}ts$ when $s = (124)(23)$ and $t = (1452)$.
9. Compute $s^{-1}ts$ when $s = (124)$ and $t = (357)$.
10. Do the Exercises on page 14.
11. Read Chapter 3 "Impossible and Imaginaries" in Pesic's text. Then give short answers to the following two questions.
12. In your own words, give a rough description of Bombelli's wild thought in Box 3.3 on page 55. In particular, show $b = 1$.

13. Explain the excerpt from page 54 "Conversely, if the root is an imaginary number, we could reject it on the grounds that such a solution is not allowable because it is not real. However, in the case of cubic equations, even when all the roots are real, the del Ferro-Cardano-Tartaglia formula explicitly involves imaginary numbers."

14. Read Chapter 4 "Spirals and Seashores" in Pesic's text. Then give short answers to the following three questions.

15. Use Girard's identities in Box 4.1 to show that $x = 5$ cannot be a solution to the quadratic $x^2 - 9x + 2$.

16. In your own words describe the passage from page 67, "...the Swedish mathematician E. S. Bring showed that we can also get rid of the x^2 term, leaving the general quintic in the much simplified form $x^5 + px + q = 0$. The quintic now looked so simple that it became increasingly perplexing why it would not yield...Leibniz wondered if an extension of Tschirnhaus's technique could simplify the quintic into the form x^5 equals a constant, which would be readily solvable."

17. Describe the relation of the Fundamental Theorem of Algebra to the goal of our course: To solve the general quintic in radicals.