Homework 2 Due Wednesday 22 February

1. Read the Introduction to Pesic's text "Abel's Proof"

2. Briefly explain what Pesic is saying in the paragraph on the bottom of page 2. Explain how we can solve the particular quintic $x^5 + 5x - 6 = 0$ by using the factor theorem but, in general, the quintic is not solvable in radicals. (HINT: you do not need to completely solve the particular quintic $x^5 - 5x - 6 = 0$ if it requires invoking the quartic or cubic formulas. Just explain how you would solve it.)

3. Find a cubic equation $ax^3 + bx^2 + cx + d = 0$ having the roots $1, 2, 3$. Is there more than one cubic having these roots? How many? How many cubics have these roots if $a = 1$?

4. Find a quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ having the roots $\pm 1, \pm 2$.

5. Use any method to solve the quartic $x^4 - 81 = 0$.

6. Find the discriminant of $x^4 - 3x^3 + x^2 + 3x - 2 = 0$ and show that the equation has a multiple root (HINT: you do not need to find the root in order to answer this part).

7. Find the coefficients of x^{40} and x^{39} in the polynomial
$$(1 + x + x^2 + x^3 + \ldots + x^{20})^2.$$

8. Find the coefficients of x^{40} and x^{39} in the polynomial
$$(1 + x + x^2 + x^3 + \ldots + x^{10})^4.$$

9. (Optional) Read Ch 1. "The Scandal of the Irrational" in Pesic's text. This material will not appear on future exams or homework in this course but it is fascinating nonetheless. With this chapter Pesic sets the stage for our search for a method to solve the general quintic equation by radicals. The solution is that there is no such method. This may seem simple at first and not worth thinking about. You could say, "OK there is no method so there is nothing remaining to say. We should move on to another problem where there are *real* solutions ..." Yet Pesic tries to convince you otherwise by giving an example of another famous (maybe more famous?) problem in mathematics where there is no solution.

10. Read section 3 in Dickson's text p.4—(top of page) 6.

11. Explain equation (9) : $\Psi_1\Psi_4 = c_1^2 - 3c_2$ on page 5 of Dickson's text.

12. In your own words re-do the computations leading from equation (10) on page 5 to the solutions (11) on page 6 in Dickson's text.

13. Read Ch 2 "Controversy and Coefficients" in Pesic's text.

14. Read Al-Kwarizmi's geometric solution to the quadratic in Box 2.3 on page 26. Use it to solve the quadratic $x^2 + 6x - 16 = 0$. (In your solution you must draw and clearly label squares and rectangles.)

15. Read Cardano's method of completing the cube in Box 2.4 on page 36. Use this method to solve: $y^3 + y - 2 = 0$.

16. Use Box 2.5 in Pesic's book (p.38-39) to write the resolvent cubic, equation (2.5.3), of the quartic, $x^4 + 2x^3 - 12x^2 - 10x + 3 = 0$. (Note: you do not have to solve the quartic; you only need to find its resolvent cubic.)