

Homework 1 Due Wednesday 8 February

This week we will read pages 1–4 of Dickson’s text (there is a link to a pdf copy on our website). Reading Dickson can be slow going at first. You may need to reread a page several times to understand it. After you have finished the reading, try the following problems. When you solve a polynomial equation, find the real and complex solutions.

1. Find all solutions of $x + 5 = 0$
2. Find all solutions of $x^2 + 5 = 0$
3. Find all solutions of $x^3 + 5 = 0$
4. Instead of memorizing the cubic formula in Dickson’s text “Introduction to the Theory of Equations”, it is probably best to memorize the two substitutions: 1. $x = y + \frac{1}{3}c_1$ into the general cubic $x^3 - c_1x^2 + c_2x - c_3 = 0$ and then 2. $y = z - \frac{p}{3z}$ into $y^3 + py = q = 0$, the result from the first substitution. (Of course, if you prefer to memorize the entire formulas you can visit the wikipedia site.) In this problem I want you to use substitutions to solve the cubic $x^3 - 3x^2 + 4x - 4 = 0$.
5. Redo the previous problem by recognizing a root is $x_1 = 2$ by trial and error. (This is how the problem would be solved in high school.)
6. Explain why the solutions you found in problems 2 and 3 look different. It is often helpful to review a simple quadratic equation when you get confused with higher degree problems.
7. Solve $x^2 - x - 6 = 0$ by factoring
8. Solve $x^2 - x - 6 = 0$ by using the quadratic formula
9. Solve $x^2 - x - 6 = 0$ by making the substitution $x = t + \frac{-b}{2a}$.
10. Using $\omega = -\frac{1}{2} + \frac{\sqrt{-3}}{2}$, show that $\omega^2 = -\frac{1}{2} - \frac{\sqrt{-3}}{2}$ and $\omega^2 + \omega + 1 = 0$.
11. Use the factor theorem to show that $x^{20} - 1 = (x - 1)p(x)$ when $p(x)$ is some polynomial. Find $p(x)$.
12. Do problems 1–6 in Dickson’s text.