DEPARTMENT OF MATHEMATICS

Math 391 · · · · · · · · · · · · · · Final Examination · · · · · · · · · · · · · · Date: May, 2005

Part I. Answer ALL questions. Total 64 points.

1. [13 Points] Solve the initial value problem:

$$y'' - 4y' + 4y = x^2 + 12e^{2x}, \qquad y(0) = 1, \ y'(0) = 0.$$

2. [8 Points] Solve

$$\left(y\cos(xy) + \frac{y}{2x}\right)dx + \left(x\cos(xy) + \frac{1}{2}\ln(x) + \frac{1}{e^y}\right)dy = 0.$$

3. [9 Points] Find the general solution to

$$y'' - 2y' + y = \frac{e^x}{x}.$$

4. [7 Points] Solve $xy' - 2y = xy + xe^x$.

5. [13 Points] For the equation $2xy'' - y' + y = 0$,

(a) Show $x = 0$ is a regular singular point.

(b) Find the indicial equation and the recurrence relation corresponding to the larger root.

(c) Find the first four terms of the series solution valid near $x > 0$ correcponding to the larger root.

6. [4 Points] Use separation of variables to replace the partial differential equation:

$$xtu_{xx} + u_{xt} + tu_x = 0,$$

where u is a function of x and t, by two ordinary differential equations.

7. [10 Points] Use the Laplace Transform method to solve:

$$y'' + 4y = 2, \qquad y(0) = 1, \ y'(0) = 3.$$

Note that: $\qquad \mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}, \qquad \mathcal{L}\{\sin at\} = \dfrac{a}{s^2+a^2} \qquad \mathcal{L}\{\cos at\} = \dfrac{s}{s^2+a^2}.$

Part II begins on the back.

Part II. Answer any THREE (3) COMPLETE questions. Total: 36 points.

8. [12 Points] Find the Fourier series for

$$f(x) = \begin{cases} x + 2 & \text{if} \quad -2 < x \le 0; \\ 2 - x & \text{if} \quad 0 < x \le 2, \end{cases}$$

where $f(x + 4) = f(x)$ for all x.

9. [12 Points] Find the terms of the power series solution through x^5 of

$$y'' - y' + xy = 0, \qquad y(0) = 1, \ y'(0) = 2.$$

10. **(a)** [4 Points] Solve $2x^2 y'' + xy' - y = 0$.

(b) [8 Points] A mass weighing two pounds stretches a spring 6 inches. The mass is pulled down 3 inches and given an upward velocity of 1 ft/sec. Find $u(t)$, the displacement of the mass in feet from its equilibrium position at time t seconds after release. Assume that the acceleration due to gravity is 32 ft/sec^2 and that air resistance is negligible.

11. A 200 gallon tank is half full of pure water. A salt solution with a concentration 5 lb/gal is flowing into the tank at the rate of 4 gal/min while the well-mixed solution is flowing out at the rate of 2 gal/min.

(a) [9 Points] Find $Q(t)$, the amount of salt in lbs in the tank at time t minutes.

(b) [3 Points] Find the concentration of salt in the tank when the tank overflows.