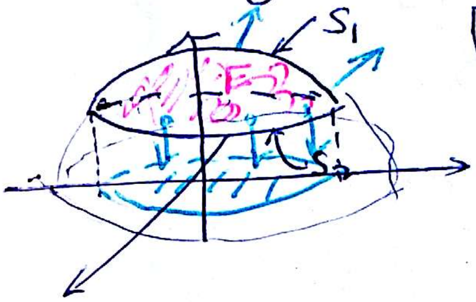
4/3 Exam 2 Review
Linear Algebra Introduction.

Review Example) Verify the Divergence theorem when E is the solid $\begin{cases} x^2+y^2+z^2 \le 4 \\ z \geq 1 \end{cases}$ S is the boundary and $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$. Use outward normal of S.



$$\boxed{\begin{array}{c} 1 = z = 2\cos\phi \\ \phi = \pi/3 \end{array}}$$

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \iint_{S_1} \langle x,y,z\rangle \cdot \underbrace{\frac{\langle x,y,z\rangle}{2}\, dS}_{d\vec{S}} = \iint_{S_1} \frac{4}{2}\, dS = 2\,\text{Area}(S_1)$$

$$= 2\int_{\theta=0}^{\theta=2\pi}\int_{\phi=0}^{\phi\,\pi/3} 2^2 \sin\phi\, d\phi\, d\theta = 2^4\pi\,(-\cos\phi)\Big|_{\phi=0}^{\phi=\pi/3} = 8\pi$$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \iint_{S_2} \underbrace{\langle x,y,z\rangle}_{\vec{F}} \cdot \underbrace{\langle 0,0,-1\rangle}_{\hat{n}}\, \underbrace{dx\,dy}_{dA} = \iint_{S_2} -\overset{-1}{\overset{\shortparallel}{z}}\, dA = -\text{Area}(S_2)$$
$$= -3\pi$$

$$\boxed{\text{when } z=1,\ x^2+y^2+1^2 = 4 \rightarrow x^2+y^2 = 3}$$

$$\oiint_{S} \vec{F}\cdot d\vec{S} = \iint_{S_1} \vec{F}\cdot d\vec{S} + \iint_{S_2} \vec{F}\cdot d\vec{S} = 8\pi + (-3\pi)$$
$$= 5\pi$$

To verify the divergence theorem $\oiint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div}\vec{F}\, dV$.

$$\operatorname{div}\vec{F} = (x)_x + (y)_y + (z)_z = 1+1+1 = 3$$

$$\iiint_E \operatorname{div}\vec{F}\, dV = \iiint_E 3\, dV = 3 \operatorname{vol}(E)$$

$$= 3 \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\sqrt{3}} \int_{z=1}^{z=\sqrt{4-\underbrace{x^2-y^2}_{-r^2}}} r\, dz\, dr\, d\theta$$

$$= 3 \cdot 2\pi \int_{r=0}^{r=\sqrt{3}} r \cdot \left(\sqrt{4-r^2} - 1\right) dr \qquad \boxed{u = 4-r^2}$$

$$= 6\pi \left[\frac{(4-r^2)^{3/2}}{-3} - \frac{r^2}{2} \right] \Bigg|_{r=0}^{r=\sqrt{3}}$$

$$= 6\pi \left(\left[-\frac{1}{3} - \frac{3}{2} \right] - \left[-\frac{8}{3} - 0 \right] \right)$$

$$= 6\pi \left(\frac{7}{3} - \frac{3}{2} \right) = 6\pi \left(\frac{14-9}{6} \right) = 5\pi$$

Linear Algebra

System of Equations $\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases}$

3 view points I. MATRIX II. ROW

III COLUMN

I. $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

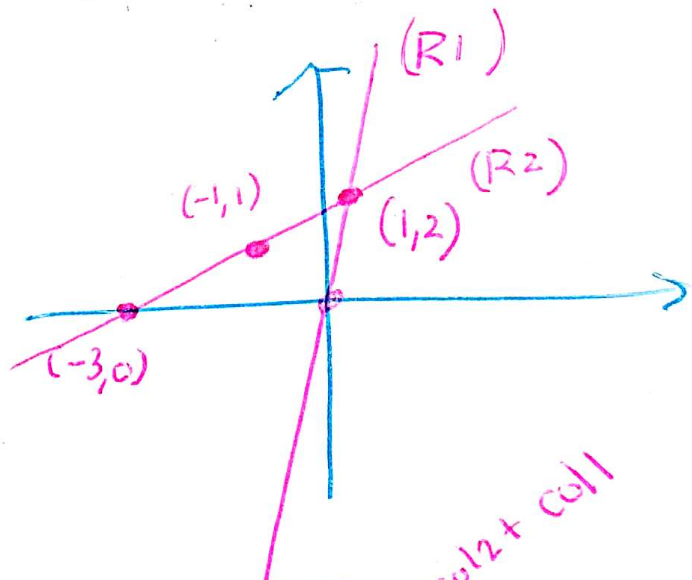
II. $2x - y = 0$ (R1)

$-x + 2y = 3$ (R2)

Intersection of two lines in plane $= \mathbb{R}^2$

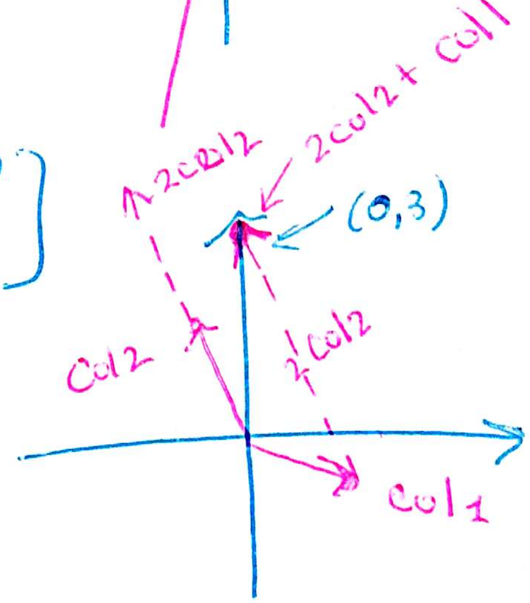
SOLUTION (1,2)

$x = 1$, $y = 2$



III $x \underbrace{\begin{bmatrix} 2 \\ -1 \end{bmatrix}}_{\text{col 1}} + y \underbrace{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}_{\text{col 2}} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

"linear combination" of columns



How to find solution (1,2)?

Answer: Augment Matrix and Gaussian Eliminate

$$\left[\begin{array}{cc|c} 2 & -1 & 0 \\ -1 & 2 & 3 \end{array}\right] \xrightarrow{GE1} \left[\begin{array}{cc|c} \boxed{1} & -2 & -3 \\ 2 & -1 & 0 \end{array}\right] \longrightarrow$$

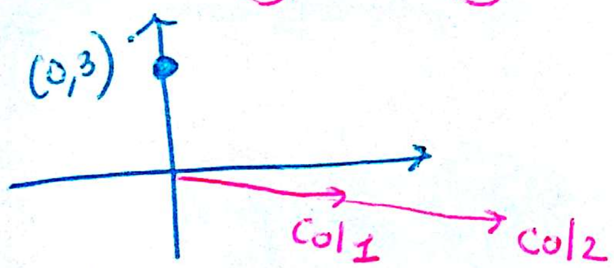
$$\xrightarrow{GE2} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 3 & 6 \end{array}\right] \quad R2 - 2R1$$

lower triangular zero. Gaussian Elimination DONE.

Rewrite system and backwards substitute

$$\begin{cases} x - 2y = -3 & \xleftarrow{\text{subst}} \quad \longrightarrow x = 1 \\ 3y = 6 & \xrightarrow[\text{bottom}]{} y = 2 \end{cases}$$

What can go wrong? what if col1 // col2



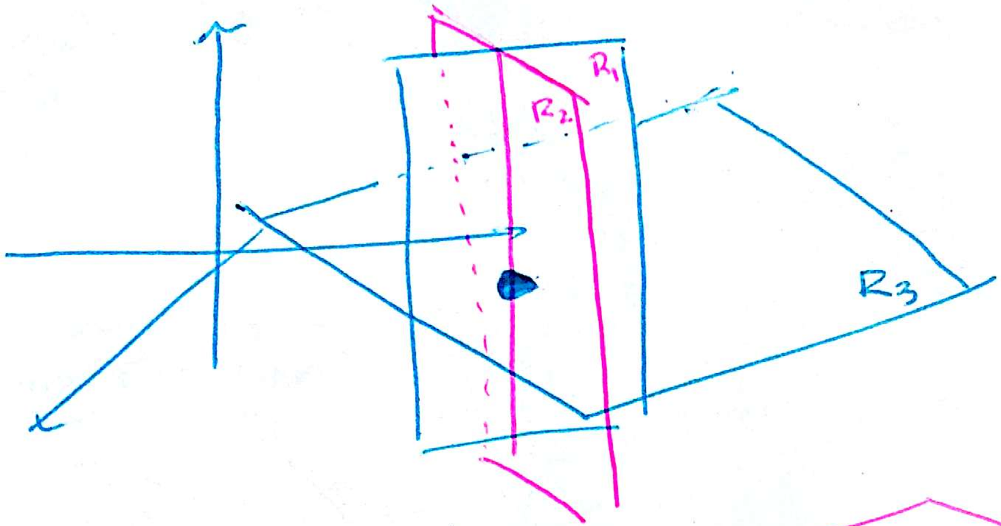
Ex2) $\begin{cases} 2x - y = 0 \ (R1) \\ -x + 2y - z = -1 \ (R2) \\ -3y + 4z = 4 \ (R3) \end{cases}$ $\hat{n}_1 = \langle 2, -1, 0 \rangle$

I. Matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

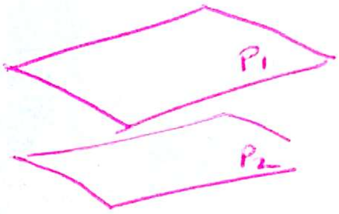
II. Row INTERSECTION OF 3 PLANES IN SPACE $= \mathbb{R}^3$.

2D



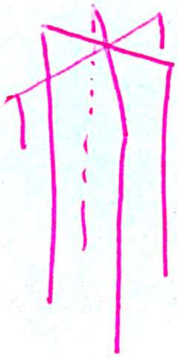
MOSTLY 3 Planes INTERSECT IN A POINT

what can go wrong? A)



Planes are Parallel
DO NOT INTERSECT

B)



3 PLANES DO NOT ALWAYS INTERSECT IN 1 POINT
IN THIS CASE, 3 PLANES INTERSECT IN A LINE.

Ex3)
$$\begin{cases} 2x_1 - 5x_2 + \frac{1}{2}x_3 + \cdots + 8x_9 = 7 \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ 11x_1 \quad + 3x_3 + \cdots + \frac{1}{2}x_9 = 10 \end{cases}$$

9 Equations 9 unknows

II Row Picture Intersection of 9 (8D) hyperplanes. Most likely one point.
in $\mathbb{R}^9$

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