Related Rates Handout

For Math 205

Related rates, as the name suggests, refers to relating the rates of change of several related quantities. As is often the case, the goal is to figure out the rate of change of one quantity if the rates of change of the other quantities are known (or can be known in theory, with *relative* ease—no pun intended, despite what the italicized text might suggest). The method of related rates is an application of implicit differentiation, which, as we've seen is an application of the chain rule. Another thing suggested by "rate" is the fact that the independent variable usually represents time, and is often denoted t. Let's get into the method and then do some examples!

The Method of Related Rates

1. **Read the problem carefully!** Read it again. Did you read the problem? Read it!
2. **If possible/necessary, draw a diagram. Label the quantities that are changing with variables and the quantities that are not changing by their constant values.** Think of the variables as functions of time.
3. **Write down the given information** in regards to the values of any rates that are known. Also **write down what you want to find**, and with what conditions. This will maintain your focus, grasshopper, as well as have the known values explicitly written down ready for use.
4. **Set up an equation that relates all the quantities under consideration.** If you drew a diagram, it will usually come in handy here. For instance, if your diagram is a triangle, the equation you come up with could be Pythogoras' theorem, the law of sines or cosines, or an application of SOHCAHTOA, etc. So the diagram would suggest what equation you would set up. Remember your geometry!
5. **Differentiate the equation in step 4 implicitly with respect to time**. This means that any variable or expression that does not explicitly involve a t will use the chain rule when differentiating (you need to multiply by the primes when differentiating them).
6. **Plug in your knowns and solve for the unknown that you seek.** At first, you may end up with several unknowns. In this case, you should be able to go back to the equation in step 4 to solve for all the unknowns you need in order to solve for the particular unknown you care about. Sometimes, you can also use the geometry suggested by the diagram to eliminate some of the variables in play to make your life easier.

Problems

1. A right circular cylinder undergoes expansion. Its radius is increasing at a rate of 1 m/s while its height is increasing at a rate of 2 m/s. At what rate is its volume changing when the radius is 1m in length and the height is 3m?

2. (a) A particle is moving along the curve $y = x^3 + 1$ in such a way that its y-coordinate is increasing at a rate of 2 units/sec. At what rate is its x-coordinate changing when $y = 9$?
(b) At this instant, how fast is the distance between the particle and the point (2,10) changing? Is the particle approaching (2,10) or getting farther from it at this point?

3. (a) A 15-foot ladder was resting against a vertical wall. Suddenly, and without warning, the foot of the ladder begins to slide away from the wall at a rate of 2 ft/s. At what rate is the top of the ladder sliding down the wall when the foot of the ladder is 9 feet from the wall?

4. At noon, ship A is 80 miles west of ship B. At this time, both ships set sail. Ship A sails south at a speed of 20 mph, while ship B sails north at a speed of 10 mph. How fast is the distance between the two ships changing at 2pm if both ships maintain constant speed?

5. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.
   (a) At what rate is his distance from second base decreasing when he is halfway to first base?
   (b) At what rate is his distance from third base increasing at the same moment?

6. Back to our story: Jhevon's llamas that he had in his backyard have bred numerously and he decided to start selling them. He sells x thousand llamas per week at a price p and figured out that x and p give the following demand curve.
$$p + 2x + xp = 38$$
   How fast are Jhevon's weekly llama sales changing at the point where he is selling 4 thousand llamas per week at a price of \$6 per llama and he is decreasing his price at a rate of \$0.40/week?

7. Superman is cruising at 390 feet per second at an altitude of 5000 feet and he flies directly over Jhevon's house. Assuming Superman maintains altitude and speed, how fast is the distance from Jhevon's house to Superman changing at the time when Superman is 13,000 feet from Jhevon's house.

P.S. There are many more interesting related rates problems in the text. The situations in Related Rates can vary widely, so I suggest you practice more than is required in the homework. You can use other texts as well.

More Examples!

1. A 5 foot ladder on level ground is leaning against a vertical wall. The foot of the ladder is being pulled along the ground at a rate of 2 meters per second. At what rate is the top of the ladder sliding down the wall when the foot of the ladder is 3 feet from the wall?

2. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

3. (From spring 2009 final, problem 11) A car must travel ten miles of inclined road to reach one mile of height. The car is traveling at a speed of 80 miles per hour. How fast is the car rising (vertical height) when the car traveled 6 miles of this road?

4. (From spring 2010 final, problem 9) Under certain conditions (called adiabatic expansion) the pressure P and the volume V of a gas satisfy the equation $P^5V^7 = 1000$. Suppose that at some moment the volume of the gas is 4 liters, the pressure is 200 units, and the pressure is increasing at a rate of 5 units per second. Find the rate at which the volume is changing.