1 Chapter 1 Fundamentals

1.1 Real Numbers

It is usually best to simplify answers as a single reduced rational expression $\frac{a}{b}$ when possible, rather write it as a mixed fraction or as a decimal. Computations in calculus will usually be simpler when you use a single rational expression. For instance it is usually preferable to write $\frac{4}{3}$ rather than $1\frac{1}{3}$ or $1.\bar{3}$.

Interval notation and inequalities are often used in calculus. You should learn how to switch between the two notations.

Exercises

1. Perform the indicated operation $1 + \frac{5}{8} - \frac{1}{6}$. Write your answer as a reduced fraction in the form $\frac{a}{b}$.

2. Perform the indicated operation $(\frac{1}{2} - \frac{1}{3})(\frac{1}{2} + \frac{1}{3})$. Write your answer as a reduced fraction in the form $\frac{a}{b}$.

3. Perform the indicated operation $\frac{2-\frac{3}{4}}{\frac{1}{2}-\frac{1}{3}}$. Write your answer as a reduced fraction in the form $\frac{a}{b}$.

4. Express the interval $(-3, 0)$ in terms of inequalities.

5. Express the interval $(-\infty, \frac{-1}{2})$ in terms of inequalities.

6. Express the inequality $-2 < x \leq \frac{7}{5}$ in interval notation.

7. Express the inequality $x > -1$ in interval notation.

1.2 Exponentials and Radicals

Most expressions like $\sqrt{30} = 30^{\frac{1}{2}}$ do not simplify as fractions. They are new numbers, irrational numbers. This is a good thing. Scientists and engineers want as many numbers as possible to describe the world. In a course like this many of these expressions will simplify as a fraction, like $\sqrt[3]{\frac{-8}{27}} = -\frac{2}{3}$. This is by design. We want you to practice using the basic laws of exponents to simplify various expressions in this course. Being able to simplify such expressions will be very helpful when you study calculus. However you should know that if the expression $\sqrt[3]{\frac{-8}{27}}$ is altered slightly to $\sqrt[3]{\frac{-7}{26}}$ then it will no longer simplify. This is a good thing: $\sqrt[3]{\frac{-7}{26}}$is a new that we can use to describe the world whereas $\sqrt[3]{\frac{-8}{27}}$ is not a new number. You already learned it in middle school as the fraction $\frac{-2}{3}$.

There is no compelling reason to eliminate negative exponents. Negative exponents can be helpful. The exercises below ask you to eliminate negative exponents because we want to see if you can do so. It is a good challenge to see if you know the laws. Later in your career at CCNY you may want to leave negative exponents in your answers. Physics and engineering books sometimes leave negative exponents in answers.

Example

Eliminate negative exponents and simplify each expression.

(a) $\dfrac{6st^{-4}}{2s^{-2}t^2}$

(b) $\left(\dfrac{y}{3z^3}\right)^{-2}$

Solution

(a) We use Law 7, which allows us to move a number raised to a power from the numerator to the denominator (or vice versa) by changing the sign of the exponent.

t^{-4} moves to denominator and becomes t^4

$$\frac{6st^{-4}}{2s^{-2}t^2} = \frac{6ss^2}{2t^2t^4} \qquad \text{Law 7}$$

s^{-2} moves to numerator and becomes s^2

$$= \frac{3s^3}{t^6} \qquad \text{Law 1}$$

(b) We use Law 6, which allows us to change the sign of the exponent of a fraction by inverting the fraction.

$$\left(\frac{y}{3z^3}\right)^{-2} = \left(\frac{3z^3}{y}\right)^2 \qquad \text{Law 6}$$

$$= \frac{9z^6}{y^2} \qquad \text{Laws 5 and 4}$$

Exercises

1. Simplify $\sqrt{64}$ when possible.
2. Simplify $\sqrt[3]{-64}$ when possible.
3. Simplify $\sqrt[3]{51}$ when possible.
4. Simplify $\sqrt[5]{-32}$ when possible.
5. Simplify $\sqrt[3]{\frac{8}{27}}$ when possible.
6. Simplify $\frac{\sqrt{48}}{\sqrt{3}}$ when possible.
7. Simplify $\frac{4}{9}^{-\frac{1}{2}}$ when possible.

8. Simplify $32^{-\frac{2}{5}}$ when possible.
9. Simplify $17^{\frac{2}{3}}$ when possible.
10. Simplify $(rs)^3(2s)^{-2}(4r)^4$. Eliminate negative exponents.
11. Simplify $(2u^2v^3)^3(8u^3v)^{-2}$. Eliminate negative exponents.
12. Simplify $\frac{(x^2y^3)(xy^4)^{-3}}{x^2y}$. Eliminate negative exponents.
13. Simplify ${\frac{(9st)^{\frac{3}{2}}}{81s^3t^{-1}}}^{\frac{1}{2}}$. Eliminate negative exponents.

1.3 Algebraic Expressions

Factoring vs. Expanding. Understand the difference.

We use the Distributive Property to expand algebraic expressions. We sometimes need to reverse this process (again using the Distributive Property) by **factoring** an expression as a product of simpler ones. For example, we can write

EXPANDING

$$(x-2)(x+3) = x^2 + x - 6$$

FACTORING

We say that $x-2$ and $x+3$ are **factors** of x^2+x-6.

The easiest type of factoring occurs when the terms have a common factor.

Exercises

1. Expand $8(2x+5) - 7(x-9)$ and simplify.
2. Expand $x(2x-11) - 9(2x-10)$ and simplify.
3. Expand $(3t-2)7t-5)$ and simplify.
4. Expand $(c+\frac{1}{c})^2$ and simplify.
5. Expand $\sqrt{x}(x-\sqrt{x})$ and simplify.
6. Expand $(\sqrt{h^2+1}+1)(\sqrt{h^2+1}-1)$ and simplify.
7. Factor $x^2+2x-15$.
8. Factor $16x^2-81$ by finding a GCF.
9. Factor $2x^3+x^2-6x-3$ by grouping.

1.4 Rational Expressions

Warning

Don't make the mistake of applying properties of multiplication to the operation of addition. Many of the common errors in algebra involve doing just that. The following table states several properties of multiplication and illustrates the error in applying them to addition.

Multiplication Property	Common Error with Addition
$(a \cdot b)^2 = a^2 \cdot b^2$	$(a+b)^2 \cancel{=} a^2 + b^2$
$\sqrt{a \cdot b} = \sqrt{a}\sqrt{b} \quad (a, b \geq 0)$	$\sqrt{a+b} \cancel{=} \sqrt{a} + \sqrt{b}$
$\sqrt{a^2 \cdot b^2} = a \cdot b \quad (a, b \geq 0)$	$\sqrt{a^2+b^2} \cancel{=} a + b$
$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$	$\frac{1}{a} + \frac{1}{b} \cancel{=} \frac{1}{a+b}$
$\frac{ab}{a} = b$	$\frac{a+b}{a} \cancel{=} b$
$(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$	$(a+b)^{-1} \cancel{=} a^{-1} + b^{-1}$

To verify that the equations in the right-hand column are wrong, simply substitute numbers for *a* and *b* and calculate each side. For example, if we take $a = 2$ and $b = 2$ in the fourth error, we get different values for the left- and right-hand sides:

$$\underbrace{\frac{1}{a} + \frac{1}{b} = \frac{1}{2} + \frac{1}{2} = 1}_{\text{Left-hand side}} \qquad \underbrace{\frac{1}{a+b} = \frac{1}{2+2} = \frac{1}{4}}_{\text{Right-hand side}}$$

Exercises

1. Find the domain of $-x^4 + x^3 + 9x$.
2. Find the domain of $\frac{2t^2-5}{3t+6}$.
3. Find the domain of $\sqrt{2t+3}$.
4. Simplify $\frac{x^2-x-2}{x^2-1}$.
5. Simplify $\frac{x^2-x-12}{x^2+5x+6}$.
6. Perform the multiplication $\frac{4x}{x^2-4} \cdot \frac{x+2}{16x}$ and simplify.

7. Perform the multiplication $\frac{t^2-t-6}{t^2+2t} \cdot \frac{t^3+t^2}{t^2-2t-3}$ and simplify.

8. Perform the division $\frac{t-3}{t^2+9} \div \frac{t^2-9}{2t+6}$ and simplify.

9. Perform the addition $2 + \frac{x}{x+3}$ and simplify.

10. Perform the substraction $\frac{x}{x-4} - \frac{3}{x+6}$ and simplify.

11. Perform the addition $\frac{1}{x+3} + \frac{1}{x^2-9}$ and simplify.

1.5 Equations

Notice the difference between the exercises in this section compared with the previous two sections. In the previous two sections we simplified algebraic and rational expressions. In this section we will *solve* algebraic and rational expressions. Understand the difference between simplifying and solving. Note that every exercise in this section has an "=" sign whereas the exercises in the previous two sections had none. The answer to the problems in the previous two sections were simplified algebraic and rational expressions. The answers to the exercises in this section are numbers that solve algebraic and rational expressions. We use the simplifying techniques learned in the previous two sections to aid us in solving equations here. Often on exams students get nervous and mistakenly write numbers for answers to simplifying exercises and simplified or factored expressions to equations. Do not make this mistake. If there is an "=" solve the equation.

Example

Find all real solutions of the equation $x^2 + 5x = 24$.

Solution

We must first rewrite the equation so that the right-hand side is 0.

$$\begin{array}{rrcll} & x^2+5x & = & 24 & \\ & x^2+5x-24 & = & 0 & \text{Subtract 24} \\ & (x-3)(x+8) & = & 0 & \text{Factor} \\ x-3=0 & \text{or} \quad x+8 & = & 0 & \text{Zero-Product Property} \\ x=3 & x & = & -8 & \text{Solve} \end{array}$$

The solutions are $x = 3$ and $x = -8$.

Exercises

1. Solve $\frac{z}{5} = \frac{3}{10}z + 7$.
2. Solve $\frac{2x-1}{x+2} = \frac{4}{5}$.
3. Solve $PV = nRT$ for R.
4. Solve $V = \frac{1}{3}\pi r^2 h$ for r.
5. Solve $x^2 + 8 = -12$.
6. Solve $x^2 + 2 = 4x$.
7. Solve $w^2 = 3(w - 1)$.
8. Solve $\frac{1}{x} = \frac{4}{3x} + 1$.
9. Solve $\frac{x^2}{x+100} = 50$.
10. Solve $\sqrt{2x+1} + 1 = x$.
11. Solve $|x - 4| = 0.01$

1.8 Inequalities

Guidelines for Solving Nonlinear Inequalities

1. **Move All Terms to One Side.** If necessary, rewrite the inequality so that all nonzero terms appear on one side of the inequality sign. If the nonzero side of the inequality involves quotients, bring them to a common denominator.
2. **Factor.** Factor the nonzero side of the inequality.
3. **Find the Intervals.** Determine the values for which each factor is zero. These numbers divide the real line into intervals. List the intervals that are determined by these numbers.
4. **Make a Table or Diagram.** Use **test values** to make a table or diagram of the signs of each factor on each interval. In the last row of the table determine the sign of the product (or quotient) of these factors.
5. **Solve.** Use the sign table to find the intervals on which the inequality is satisfied. Check whether the **endpoints** of these intervals satisfy the inequality. (This may happen if the inequality involves $\leq$ or $\geq$.)

The factoring technique that is described in these guidelines works only if all nonzero terms appear on one side of the inequality symbol. If the inequality is not written in this form, first rewrite it, as indicated in Step 1.

Exercises

1. Solve $\frac{2}{5}x + 1 < \frac{1}{5} - 2x$. Express your solution in interval notation.
2. Solve $x^2 \leq x + 2$. Express your solution in interval notation.
3. Solve $x^3 < 9x$. Express your solution in interval notation.
4. Solve $\frac{2x+6}{x-2} < 0$. Express your answer using interval notation.
5. Solve $\frac{2x+1}{x-5} \leq 3$. Express your solution in interval notation.
6. Solve $|x + 1| \geq 1$. Express your solution in interval notation.
7. Solve $|5x - 2| < 6$. Express your solution in interval notation.

1.9 The Coordinate Plane; Graphs of Equations; Circles

It is important that you learn to sketch the graph of an equation by (1) making a table of values, (2) plotting the points from the table and then (3) connecting the dots to make a graph.

Example

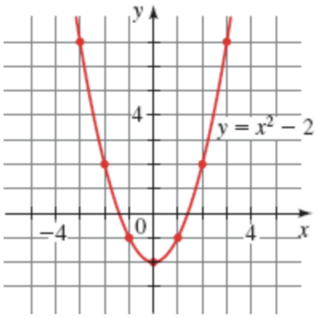
Sketch the graph of the equation $y = x^2 - 2$.

> Detailed discussions of parabolas and their geometric properties are presented in Sections 3.1 and 10.1.

Solution

We find some of the ordered pairs (x, y) that satisfy the equation in the table on the next page. In Figure 9 we plot the points corresponding to these ordered pairs and then connect them by a smooth curve. A curve with this shape is called a *parabola*.

Figure 9



x	$y = x^2 - 2$	(x, y)
−3	7	(−3, 7)
−2	2	(−2, 2)
−1	−1	(−1, −1)
0	−2	(0, −2)
1	−1	(1, −1)
2	2	(2, 2)
3	7	(3, 7)

Exercises

1. Sketch the graph of $y = x^2 - 9$ by making a table of values.

2. Sketch the graph of $y = 9 - x^2$ by making a table of values.
3. Sketch the graph of $y = \sqrt{x+4}$ by making a table of values.
4. Sketch the graph of $y = \sqrt{4 - x^2}$ by making a table of values.
5. Sketch the graph of $y = 4 - |x|$ by making a table of values.
6. Find an equation of the circle with center $(-1, -4)$ and passing through the point $(4, 2)$.
7. Find the center and radius of the circle with equation $x^2 + y^2 + 6y + 2 = 0$.

1.10 Lines

You will use lines in calculus to approximate other functions. Therefore you must be comforable with the following concepts before you take calculus.

Concepts to know well.

- The Slope of a Line
- Point-Slope Form of the Equation of a Line
- Slope-Intercept Form of the Equation of a Line
- Vertical and Horizontal Lines
- General Equation of a Line
- Parallel and Perpendicular Lines

Exercises

1. Find an equation of the line passing through $(-1, 5)$ with slopt $-\frac{7}{2}$.
2. Find an equation of the line passing through the points $(-1, -2)$ and $(4, 3)$.
3. Find and equation of the line passing through the point $(1, -6)$ and parallel to the line $x + 2y = 6$.
4. Find an equation of the line with y-intercept 6 and perpendicular to the line $2x+3y+4 = 0$.

2 Functions

2.1 Functions

Four Ways to Represent a Function

Verbal Using words:

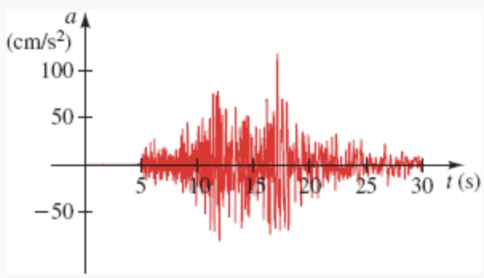
"To convert from Celsius to Fahrenheit, multiply the Celsius temperature by $\frac{9}{5}$, then add 32."

Relation between Celsius and Fahrenheit temperature scales

Algebraic Using a formula: $A(r) = \pi r^2$

Area of a circle

Visual Using a graph:



Source: California Department of Mines and Geology

Vertical acceleration during an earthquake

Numerical Using a table of values:

w (ounces)	$C(w)$ (dollars)
$0 < w \le 1$	\$0.98
$1 < w \le 2$	\$1.19
$2 < w \le 3$	\$1.40
$3 < w \le 4$	\$1.61
$4 < w \le 5$	\$1.82
$\vdots$	$\vdots$

Cost of mailing a large first-class envelope

Exercises

1. Evaluate $h(-\frac{1}{10})$ when $h(x) = x + \frac{5}{x}$.
2. Evaluate $f(x+2)$ when $f(x) = 1 - x^2$.
3. Evaluate $f(\frac{t}{3})$ when $g(t) = 18t - 13$.
4. Evaluate $f(2) - f(-2)$ when

$$f(x) = \begin{cases} 2x & \text{if } x \le -1, \\ -(1+x)^2 & \text{if } x > -1. \end{cases}$$

5. Evaluate and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$ when $f(x) = 2 - x^2$.

6. Evaluate and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$ when $f(x)=\frac{2}{x}$.
7. Find the domain $h(t)=\frac{x+2}{x^2-1}$.
8. Find the domain $g(x)=\sqrt{10-x}$.

2.2 Graphs of Functions

It is important that you can quickly sketch a rough graph of a function by making a table of values.

Exercises

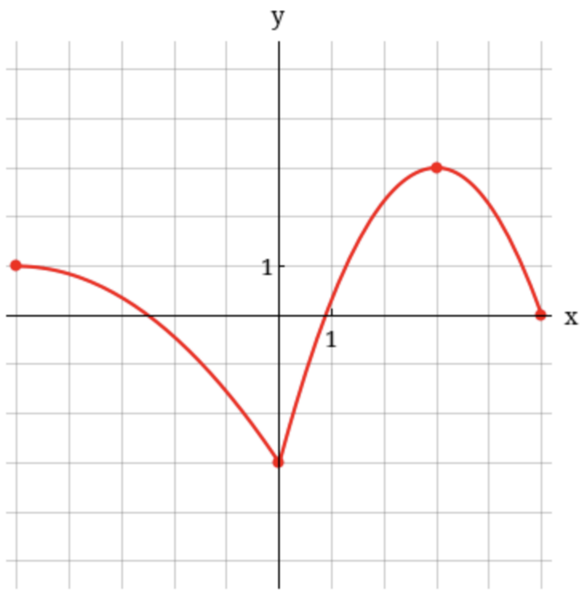
1. Sketch the graph of $f(x)=6-3x$ by making a table of values.
2. Sketch the graph of $f(x)=\sqrt{x+4}$ by making a table of values.
3. Sketch the graph of $f(x)=\frac{1}{x+4}$ by making a table of values.
4. Sketch the graph of $f(x)=\frac{|x|}{x}$ by making a table of values.
5. Sketch the graph of

$$f(x)=\begin{cases} 2 & \text{if } x \leq -1, \\ 5-x^2 & \text{if } x > -1. \end{cases}$$

2.3 Getting Information from the Graph of a Function

There are some simple conclusions you can make from looking at the graph of a function. If you find this section to be too easy, it is. In calculus you'll be asked to answer the same questions without the graph. It is important that you understand the following four concepts.

- Values of a Function; Domain and Range
- Comparing Function Values: Solving Equations and Inequalities Graphically
- Increasing and Decreasing Functions
- Local Maximum and Minimum Values of a Function



Each of the below exercises refers the following graph of a function $y = f(x)$.

Exercises

1. Find the domain of f.
2. Find the domain of f.
3. Find the interval(s) on which f is increasing.
4. Find the interval(s) on which f is decreasing.
5. Find the local maximum value(s) of f and the x value at which it occurs.
6. Find the local minimum value(s) of f and the x value at which it occurs.

2.4 Net Change and Average Rate of Change

Calculus provides tools to study the *net change* and the *instantaneous rate of change* of functions. We will not study the instantaneous rate of change in this course. You'll have to wait for calculus. In this course it is important you can compute the net change and the *average rate of change* of a function.

Example

Calculating the Average Rate of Change

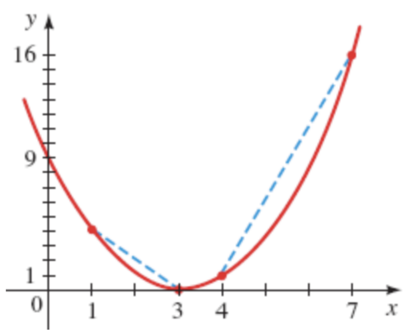
For the function $f(x) = (x-3)^2$, whose graph is shown in Figure 2, find the net change and the average rate of change between the following values of *x*.

(a) $x = 1$ and $x = 3$

(b) $x = 4$ and $x = 7$

Figure 2

$f(x) = (x-3)^2$



Solution

(a) $\text{Net change} = f(3) - f(1)$ Definition

$= (3-3)^2 - (1-3)^2$ Use $f(x) = (x-3)^2$

$= -4$ Calculate

$$\text{Average rate of change} = \frac{f(3) - f(1)}{3-1} \quad \text{Definition}$$

$$= \frac{-4}{2} = -2 \quad \text{Calculate}$$

(b) $\text{Net change} = f(7) - f(4)$ Definition

$= (7-3)^2 - (4-3)^2$ Use $f(x) = (x-3)^2$

$= 15$ Calculate

$$\text{Average rate of change} = \frac{f(7) - f(4)}{7-4} \quad \text{Definition}$$

$$= \frac{15}{3} = 5 \quad \text{Calculate}$$

Exercises

1. Determine the (a) net change and (b) the average rate of change of $f(x) = 3x - 2$ between $x = -2$ and $x = 3$.

2. Determine the (a) net change and (b) the average rate of change of $f(t) = t^2 + 2t$ between $t = -1$ and $t = 4$.

3. Determine the (a) net change and (b) the average rate of change of $g(x) = 3x^2$ between $x = 2$ and $x = 2 + h$.

4. Determine the (a) net change and (b) the average rate of change of $f(x) = 5 - x^2$ between $x = -1$ and $x = -1 + h$.

5. Determine the (a) net change and (b) the average rate of change of $g(t) = \frac{7}{t}$ between $t = a$ and $t = a + h$.

2.6 Transformations of Functions

Computers and calculators can quickly and accurately graph functions by plotting points. Humans struggle to make large tables of function values accurately. It is often simpler for us to sketch graphs using the transformation technique introduced in this section. The transformation technique at first seems wildly complicated but once you master it, you'll always want to use it.

Unfortunately the transformation technique does not always work. The transformation technique only works when the graph you want is the tranformed graph of a basic graph whose shape you've already memorized. Later in the course when we graph $y = \sin x$ and $y = \cos x$ for the first time, you won't be able to use the transformation method to graph them. You'll have to make a table. The table method always works. After you've made a table and graphed $y = \sin x$ and $y = \cos x$, you store those shapes in your file of basic shapes. You can then use the transformation method to graph all types of trigonometric functions like $y = f(t) = -5\sin(\pi t - \frac{\pi}{3})$. You will not want to make trigonometric tables after you've done it once.

Exercises

1. Use the transformation method to sketch the graph of $f(x) = (x - 12)^2$.

2. Use the transformation method to sketch the graph of $f(x) = |x + 10| - 3$.

3. Use the transformation method to sketch the graph of $f(x) = 2 - \sqrt{x + 9}$.

4. Use the transformation method to sketch the graph of $f(x) = 10 - (x - 7)^3$.

2.7 Combining Functions

We will focus on composition of functions in this section.

Exercises

1. Evaluate $f(g(5)) - g(f(0))$ when $f(x) = 1-2x$ and $g(x) = x^2-5$. Simplify your answer.
2. Evaluate $f \circ f(10) + g \circ g(2)$ when $f(x) = 1-2x$ and $g(x) = x^2-1$. Simplify your answer.
3. Find $f \circ g(x)$ when $f(x) = 1-x^2$ and $g(x) = x-2$.
4. Find $g(f(x))$ when $f(x) = 1-x^2$ and $g(x) = x-2$.
5. Find $f \circ g \circ h$ when $f(x) = \frac{3}{x}$, $g(x) = \sqrt{x+1}$, and $h(x) = x^2+2$.

2.8 One-to-One Functions and Their Inverses

Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \quad \Leftrightarrow \quad f(x) = y$$

for any y in B.

You can always convert $f^{-1}(y) = x$ to $y = f(x)$. They are the same thing. In the same way you can convert the function table $y = f(x)$ to $f^{-1}(y) = x$. You just flip the $x-$ column with the $y-$ column. Learning inverses allows us to, in some sense, double our library of functions. For every function that passes the horizontal line test we get an inverse function. Since one of the goals of applications of college math is to learn as mnay functions as possible to describe the world, learning about inverses is very helpful.

Exercises

1. Evaluate $f^{-1}(19)$ when $f(x) = 1-2x$.
2. Evaluate $f^{-1}(10)$ when $f(x) = 1-x^3$.
3. Find $f^{-1}(x)$ when $f(x) = 1-x^3$.
4. Find $f^{-1}(x)$ when $f(x) = 10+2x$.
5. Find $f^{-1}(x)$ when $f(x) = \frac{1+x}{2-x}$.

3 Polynomial and Rational Functions

3.1 Quadratic Functions

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **vertex form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with vertex (h, k).

If $a > 0$ the parabola **opens upward**

If $a < 0$ the parabola **opens downward**

The goal of this section is to graph quadratic functions $f(x) = ax^2 + bx + c$ and then analyze the graph by determining the quadratic function's domain, range, intercepts, maximum, minimum, as well as its intervals of increase and decrease. One way to do this is to complete the square to convert the quadratic $f(x) = ax^2 + bx + c$ into vertex form $f(x) = a(x-h)^2 + k$. Most students prefer instead to memorize the vertex formula $x = \frac{-b}{2a}$, which ironically only gives you the $x-$component of the vertex. Most students prefer to avoid completing the square whenever possible.

Exercises

1. Sketch the graph of $f(x) = 2x^2 - 12x + 23$ and find its vertex and its domain.

2. Sketch the graph of $f(x) = -x^2 + x + 2$ and find its vertex and its range.

3. Sketch the graph of $f(x) = x^2 + 4x$ and find its vertex and intercepts.

4. Sketch the graph of $f(x) = 1 - 6x - x^2$ and find its vertex and interval of increase.

5. Find the minimum value of $f(x) = 5x^2 - 30x + 49$.

6. If a ball is thrown directly upward with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. What is the maximum height of the ball?

3.2 Polynomial Functions and Their Graphs

Guidelines for Graphing Polynomial Functions

1. **Zeros**. Factor the polynomial to find all its real zeros: these are the x-intercepts of the graph.
2. **Test Points**. Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the x-axis on the intervals determined by the zeros. Include the y-intercept in the table.
3. **End Behavior**. Determine the end behavior of the polynomial.
4. **Graph**. Plot the intercepts and other points you found in the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

The method of graphing a polynomial function by finding its $x-$intercepts and then using test points between the intercepts does not always work. The method only works if you can find the zeros of the polynomial which is often impossible. Degree one polynomials of the form $f(x) = mx + b$ should be graphed by making a table with two points, since it is a line. Quadratic polynomials $f(x) = ax^2 + bx + c$ should be graphed using the techniques from section 3.1. Polynomials of degree more than two like $f(x) = x^5 - x^3$ should be graphed using the zeros method by following the fours steps above.

Exercises

1. Sketch the graph of $f(x) = x(x - 3)(x + 2)$. State its end behavior.
2. Sketch the graph of $f(x) = (x - 1)^2(x - 3)$. State its end behavior.
3. Sketch the graph of $f(x) = -x^3 + x^2 + 12x$. State its end behavior.
4. Sketch the graph of $f(x) = x^4 - 3x^3 + 2x^2$. State its end behavior.
5. Sketch the graph of $f(x) = x^5 - 9x^3$. State its end behavior.