MATH 392 QUIZ 1 - Version B Answers

1. Write the general form for $\int\int\int f(x,y,z)\,dV$ in:

(a) Cylindrical coordinates: $\int\int\int f(r\cos\theta, r\sin\theta, z) r\; dz\; dr\; d\theta$

(b) Spherical coordinates: $\int\int\int f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\; d\rho\; d\phi\; d\theta$

2. State the formula for the following, defining what the symbols/variables mean:

(a) a line (3 forms):

formula 1: $< x, y, z >=< x_0, y_0, z_0 > +t < a, b, c >$

formula 2: $x = x_0 + at,\; y = y_0 + bt,\; z = z_0 + ct$

formula 3: $\dfrac{x - x_0}{a} = \dfrac{y = y_0}{b} = \dfrac{z - z_0}{c}$

Meanings: (x_0, y_0, z_0) – – point on line, $< a, b, c >$ – – direction vector of line

(b) a plane: formula: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Meanings: (x_0, y_0, z_0) – – point in plane, $< a, b, c >$ – – normal vector to plane

(c) the tangent plane to the surface $F(x, y, z) = k$ at the point (a, b, c):

formula: $F_x(x - a) + F_y(y - b) + F_z(z - c) = 0$

Meanings: F_x, F_y, F_z – –partial derivatives of F evaluated at (a, b, c)

3. Compute:

(a) $\langle -1, 2, 0\rangle \times \langle 3, 4, -2\rangle$ $< -4, -2, -10 >$

(b) $\langle \pi, -3\cos t, 4t^2\rangle \cdot \langle 2, e^t, 2t^{-2}\sin t^2\rangle$ $2\pi - 3e^t\cos t + 8\sin t^2$

4. / 5. Set up and compute a triple integral to compute the volume of the region bounded by $z = \sqrt{x^2 + y^2}$ and $z = 4$ in the first octant. Include a sketch in your answer.

$V = \int_0^{\pi/2}\int_0^4\int_r^4 r\; dz\; dr\; d\theta = \dfrac{16\pi}{3}$