MATH 392 QUIZ 1 - Version A Answers

1. State the formula for the following, defining what the symbols/variables mean:
(a) a line (3 forms):
formula 1: $< x, y, z >=< x_0, y_0, z_0 > +t < a, b, c >$
formula 2: $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$
formula 3: $\dfrac{x - x_0}{a} = \dfrac{y = y_0}{b} = \dfrac{z - z_0}{c}$
Meanings: (x_0, y_0, z_0) – – point on line, $< a, b, c >$ – – direction vector of line

(b) the tangent plane to the surface $F(x, y, z) = k$ at the point (a, b, c):
formula: $F_x(x - a) + F_y(y - b) + F_z(z - c) = 0$
Meanings: F_x, F_y, F_z – –partial derivatives of F evaluated at (a, b, c)

(c) a plane: formula: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
Meanings: (x_0, y_0, z_0) – – point in plane, $< a, b, c >$ – – normal vector to plane

2. Write the general form for $\iiint f(x, y, z)\ dV$ in:
(a) Cylindrical coordinates: $\iiint f(r\cos\theta, r\sin\theta, z) r\ dz\ dr\ d\theta$
(b) Spherical coordinates: $\iiint f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\ d\rho\ d\phi\ d\theta$

3. Compute:
(a) $\langle 1, 0, 3\rangle \times \langle 2, -1, 7\rangle$ $< 3, -1, -1 >$

(b) $\langle 3t^2, 4\sin t, 7\rangle \cdot \langle \cos t, t - 2, 0\rangle$ $3t^2\cos t + 4(t - 2)\sin t$

4. / 5. Set up and compute a triple integral to compute the volume of the region bounded by $z = x^2 + y^2$ and $z = 4$ in the first octant. Include a sketch in your answer.

$V = \int_0^{\pi/2}\int_0^2\int_{r^2}^4 r\ dz\ dr\ d\theta = 2\pi$