Math 392 Quiz 4B

February 22, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f.

2. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

3. Let $\vec{F} = < P(x,y). Q(x,y) >$ be a vector field on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

4. Assume the following vector fields are conservative. Find their (scalar) potential functions, f:

   (a) $\vec{F} = < xy^2, x^2y > \Rightarrow f =$ $\frac{1}{2}x^2y^2 + k$, k is a constant

   (b) $\vec{F} =< e^x \cos y . -e^x \sin y , 2 > \Rightarrow f =$ $e^x \cos y + 2z + k$, k is a constant

5. Let $\vec{r}(t)$ be the line segment from $(1, \pi, 0)$ to $(0, \pi/2, 3)$.

   (a) Parametrize $\vec{r}(t)$ (include limits!) $\vec{r}(t) =$ $\langle 1-t, \pi - \frac{\pi}{2}t, 3t \rangle$, $0 \leq t \leq 1$

   (b) Find $\vec{r}'(t) = d\boldsymbol{r} =$ $\langle -1, -\frac{\pi}{2}, 3 \rangle$

   (c) Using parts (a) and (b), or otherwise, compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F}$ is given by 4(b) and C is given by 5(a).

   $6 + e$

Bonus:

1. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\, dA$

2. Describe what all the symbols mean in the equation above: P, Q are functions of x, y with continuous first partials on D. C is a piecewise smooth, closed curve which is the boundary of D. D is an open, simply connected region in $\mathbb{R}^2$.

For bonus 1: $\int_C \vec{F} \cdot d\vec{r} = \iint_D Q_x - P_y \, dA$ is also acceptable.