Math 392 Quiz 6B
March 13, 2018

Name: ANSWERS

Instructions: No calculators! Answer <u>all</u> problems in the space provided! Do your rough work on scrap paper.

1. What is the general form of the normal vector to a surface parametrized by $\vec{r}(u,v)$?

   $\vec{n} =$ $\vec{r}_u \times \vec{r}_v$

2. Find a vector function for rightward pointing normal vectors to $y = x^2 + z^2$

   $\vec{n} =$ $\pm \langle -2x, 1, -2z \rangle$

3. Let S be the open cylinder $x^2 + z^2 = 4$, $0 \le y \le 1$.

   (a) (2 points) Parametrize S (include limits!): $\vec{r}(u,v) = \langle 2\cos u, v, 2\sin u \rangle$, $0 \le u \le 2\pi$, $0 \le v \le 1$

   (b) Find a normal vector to S: $\vec{n} = \pm\langle -2\cos u, 0, -2\sin u \rangle$

4. Find a tangent plane to the surface $\vec{r}(u.v) = < u, v, u^2 - v^2 >$ at the point $(1,2,-3)$:

   $-2(x-1) + 4(y-2) + (z+3) = 0$

5. A smooth surface S is parametrized by $\vec{r}(u,v)$ for $(u,v) \in D$. Assuming S is covered once as (u,v) ranges throughout D, what is the general formula for the surface area of S "over" D?

   $A =$ $\iint_D |\vec{r}_u \times \vec{r}_v| \, dA$

   If you use any symbol not in the question, define it!

Bonus:

1. A smooth surface S is parametrized by $\vec{r}(u,v)$ for $(u,v) \in D$. Assuming S is covered once as (u,v) ranges throughout D. Let $\vec{F}$ be a vector field that is defined on S, define:

   (a) $\iint_S f(x,y,z)dS =$ $\iint_D f(\vec{r}(u,v)) \, |\vec{r}_u \times \vec{r}_v| \, dA$

   (b) $\iint_S \vec{F} \cdot d\vec{S} =$ $\iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v) \, dA$