Math 392 Quiz 6A
March 13, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. What is the general form of the normal vector to a surface parametrized by $\vec{r}(s,t)$?

   $\vec{n} =$ $\vec{r}_s \times \vec{r}_t$

2. Find a vector function for rightward pointing normal vectors to $y = -x^2 - z^2$

   $\vec{n} =$ $\pm\langle 2x, 1, 2z\rangle$

3. Let S be the open cylinder $y^2 + z^2 = 4$, $1 \le x \le 2$.

   (a) (2 points) Parametrize S (include limits!): $\vec{r}(u,v) = \langle u, 2\cos v, 2\sin v\rangle$, $0 \le v \le 2\pi$, $1 \le u \le 2$

   (b) Find a normal vector to S: $\vec{n} =$ $\pm\langle 0, 2\cos v, 2\sin v\rangle$

4. Find a tangent plane to the surface $\vec{r}(u.v) = < u, v, u^2 - v^2 >$ at the point (2,1,3):

   $-4(x-2) + 2(y-1) + (z-3) = 0$

5. A smooth surface S is parametrized by $\vec{r}(s,t)$ for $(s,t) \in D$. Assuming S is covered once as (s,t) ranges throughout D, what is the general formula for the surface area of S "over" D?

   $A =$ $\iint_D |\vec{r}_s \times \vec{r}_t| \, dA$

   If you use any symbol not in the question, define it!

Bonus:

1. A smooth surface S is parametrized by $\vec{r}(s,t)$ for $(s,t) \in D$. Assuming S is covered once as (s,t) ranges throughout D. Let $\vec{F}$ be a vector field that is defined on S, define:

   (a) $\iint_S f(x,y,z)dS =$ $\iint_D f(\vec{r}(s,t)) \, |\vec{r}_s \times \vec{r}_t| \, dA$

   (b) $\iint_S \vec{F} \cdot d\vec{S} =$ $\iint_D \vec{F}(\vec{r}(s,t)) \cdot (\vec{r}_s \times \vec{r}_t) \, dA$