Math 392 Quiz 5B

February 25, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x,y)\,dy = \int_a^b f(x(t),y(t))\,y'(t)\,dt$

(b) $\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt$

(c) $\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

(where C is a smooth curve parametrized by $\vec{r}(t) =< x(t), y(t) >$. No shorthand, flesh out full definition.)

2. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f.

3. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

4. Let $\vec{F} =< P(x,y). Q(x,y) >$ be a vector field on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

5. (a) Assume $f(x,y,z) = y\cos z + x^3 y$. Compute $\nabla f = \langle 3x^2y,\ \cos z + x^3,\ -y\sin z\rangle$

(b) Let C be the piece-wise smooth curve defined by: The line segment from (1,1,0) to (1,0,0), followed by the circular arc from (1,0,0) to (-1,0,0), then followed by another line segment from (-1,0,0) to (-2, 1, $\pi/2$). Compute:

$$\int_C \nabla f\cdot d\vec{r} = f(-2,1,\tfrac{\pi}{2}) - f(1,1,0) = -10$$

where $\vec{r}(t)$ is your parametrization of C.

Bonus:

1. The field $\vec{F} =< -yz\ ,\ e^z - xz\ ,\ ye^z - xy + 2 >$ has a (scalar) potential function $f(x,y,z)$. You do not need to verify this. Find the general form of this potential function.

$f(x,y,z) = ye^z - xyz + 2z + C$

2. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

3. Describe what all the symbols mean in the equation above: C is a piece-wise smooth, positively oriented, closed curve. D is the region bounded by C. P, Q are functions of x,y defined on an open region containing D with continuous partial derivatives there.