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Math 392 Quiz 5A

March 6, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$

2. Suppose $\vec{F} = \langle P(x,y), Q(x,y) \rangle$, define div$\vec{F}$ = $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = P_x + Q_y$

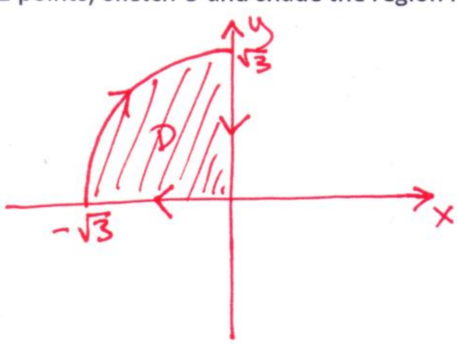
3. Find the curl and divergence of $\vec{F} = \langle x^2 + yz,\ y^2 + xz,\ z^2 + xy \rangle$.

   (a) curl$\vec{F}$ = $\langle 0, 0, 0 \rangle$ or $\vec{0}$

   (b) div$\vec{F}$ = $2x + 2y + 2z$ or $2(x+y+z)$

4. Using Green's theorem, compute $I = \oint_C 3y\,dx - 4x\,dy$, where C is the boundary of the quarter-disc $x^2 + y^2 \leq 3$ in the second quadrant, oriented *clockwise* by doing the following:

   (a) (2 points) Sketch C and shade the region it encloses.



   (b) (2 points) Set up a double integral to compute I. I = $-\iint_D (-7)\,dA = \int_{\frac{\pi}{2}}^{\pi} \int_0^{\sqrt{3}} 7\, r\,dr\,d\theta$

   (c) (2points) Evaluate the integral set up in part (b): I = $21\pi/4$

Bonus: A surface is parametrized by $\vec{r}(u,v) = \langle x(u,v),\ y(u,v),\ z(u,v) \rangle$. Give a formula to compute a normal vector to the surface at any point (u,v).

$\vec{n}$ = $\vec{r}_u \times \vec{r}_v$

An alternative answer to problem 1: $\int_C \vec{F} \cdot d\vec{r} = \iint_D (Q_x - P_y)\,dA$