Math 392 Quiz 5A

July 9, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

In this quiz, the less shorthand the better. For example, when writing a formula for which you need a normal vector $\vec{n}$, don't just write "$\vec{n}$", but rather the formula used to find it. Everything is positively oriented.

1. Let S_1 be a surface given by $z = g(x, y)$. Find a formula for a normal vector $\vec{n}_1$ to S_1: $\vec{n}_1 =$ $\pm \langle -g_x, -g_y, 1 \rangle$

2. Let S_2 be a surface parametrized by $\vec{r}(s, t)$. Find a formula for a normal vector $\vec{n}_2$ to S_2: $\vec{n}_2 =$ $\pm \vec{r}_s \times \vec{r}_t$

3. For S_1 above, define $\iint\limits_{S_1} \vec{F}(x, y, z) \cdot d\vec{S} =$ $\iint\limits_D \vec{F}(x, y, g(x,y)) \cdot \langle -g_x, -g_y, 1 \rangle \, dA$

4. For S_2 above, define $\iint\limits_{S_2} \vec{F}(x, y, z) \cdot d\vec{S} =$ $\iint\limits_D \vec{F}(\vec{r}(s,t)) \cdot (\vec{r}_s \times \vec{r}_t) \, dA$

5. State the equation in Stokes' Theorem: $\int_C \vec{F} \cdot d\vec{r} = \iint\limits_S \operatorname{curl} \vec{F} \cdot d\vec{S}$

6. Describe what the symbols above are and how they relate to each other: S is a smooth surface w/ boundary curve C. C is closed, piecewise smooth.

7. State the equation in the Divergence Theorem: $\iint\limits_S \vec{F} \cdot d\vec{S} = \iiint\limits_E \operatorname{div} \vec{F} \, dV$

8. Describe what the symbols above are and how they relate to each other: E is a solid, S is the simple closed boundary of E.

9. Let $\vec{F} = < x, y, xyz >$. Let S be the part of $z = x^2 + y^2$ that is below $z = 4$. Let C be the boundary curve of S. Fully set-up two integrals to compute the work done by $\vec{F}$ in moving a particle around C counter-clockwise.

   (a) Line integral: $\int_0^{2\pi} 0 \, dt$

   (b) Double integral: $\int_0^{2\pi} \int_0^2 0 \cdot r \, dr \, d\theta$

10. Let $\vec{F} = < x, y, xyz >$. Let E be the region bounded by $z = x^2 + y^2$ and $z = 4$. Let S be the boundary of E. Fully set-up two integrals to compute the flux out of the surface of E.

    (a) Surface integral: $\int_0^{2\pi} \int_0^2 4r^3 \cos\theta \sin\theta + r^3(2 - r^2 \cos\theta \sin\theta) \, dr \, d\theta$

    (b) Triple integral: $\int_0^{2\pi} \int_0^2 \int_{r^2}^4 (2 + r^2 \cos\theta \sin\theta) r \, dz \, dr \, d\theta$