Math 392 Quiz 3B

June 18, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f

2. Define $\int_C f(x,y,z)dy =$ $\int_a^b f(x(t),y(t),z(t))\, y'(t)\, dt$

3. Define $\int_C \vec{F} \cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

4. Let $\vec{F}$ be a vector field whose components have continuous first and second partials. What equation would you check to determine if $\vec{F}$ is conservative in the following cases?

(a) $\vec{F} = < P(x,y,z), Q(x,y,z),\ R(x,y,z) >$; equation to check: $\text{curl}\vec{F} = \vec{0}$.

(b) $\vec{F} = < P(x,y), Q(x,y) >$; equation to check: $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

5. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

6. State the equation in Green's Theorem: $\int_C P dx + Q dy = \iint_D Q_x - P_y\, dA$

7. For us, what is the most important interpretation of $\int_C \vec{F} \cdot d\vec{r}$? Work

8. Find a scalar potential f for the function $\vec{F} =< yz^2, \tan^{-1} z + xz^2, \frac{y}{1+z^2} + 2xyz >$:

$f =$ $y\tan^{-1}z + xyz^2 + C$

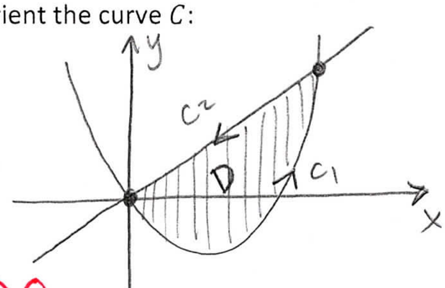
9. Let D be the region in the plane bounded by $x = y$ and $y = x^2 - x$. Let C be the positively oriented boundary of D.

Set-up integrals to compute (where a sum of integrals may be necessary): $\int_C x^3y^2dx + \frac{1}{2}x^4y\, dy$

(a) Line integral(s): $\int_0^2 t^3(t^2-t)^2 + \frac{1}{2}t^4(t^2-t)(2t-1)\, dt - \int_0^2 \frac{3}{2}t^5\, dt$

(b) Double integral(s): $\int_0^2 \int_{x^2-x}^{x} 0\, dy\, dx$

(c) Sketch the region below and orient the curve C:



Bonus:

1. Define $div\, \vec{F}(x,y) =$ $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$ where $\vec{F} = \langle P, Q \rangle$

2. If $curl\vec{F} = \vec{0}$, then $\vec{F}$ is called Irrotational; if $div\, \vec{F} = 0$, then $\vec{F}$ is called Incompressible

3. What does it mean to say "$\vec{G}$ is a vector potential of $\vec{F}$"? $\text{curl}\, \vec{G} = \vec{F}$