MATH 392 Quiz 2B

February 6, 2018

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

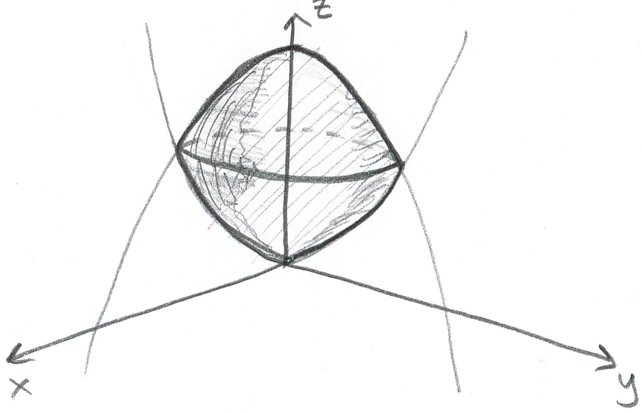
1. Let $\vec{r}(t) =< x(t), y(t), z(t) >$, $f(x, y, z)$ be a scalar function, and $P(x_1, y_1)$ and $Q(x_2, y_2)$ be points in $\mathbb{R}^2$. Complete the following rules with vector functions:

   (a) $\vec{r}\,'(t) =$ $\langle x'(t), y'(t), z'(t) \rangle$

   (b) $\nabla f =$ $\langle f_x, f_y, f_z \rangle$

   (c) Line segment $\overrightarrow{PQ} =$ $\langle x_1 + (x_2 - x_1)t,\ y_1 + (y_2 - y_1)t \rangle,\ 0 \le t \le 1$ (include limits)

2. (a) (2 points) Sketch the region bounded by $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.



   (b) Parametrize the curve of intersection, $\vec{r}_i(t)$, of the above two surfaces. Set up the limits so that the curve is traversed once.

   $\vec{r}_i(t) =$ $\langle 2\cos t, 2\sin t, 4 \rangle$ Limits: $0 \le t \le 2\pi$

3. (a) Parametrize the line segment from (-1,1,2) to (2,2,-3): $\vec{r}_l(t) =$ $\langle -1+3t, 1+t, 2-5t \rangle,\ 0 \le t \le 1$

   (b) What is the length of the above line? $L =$ $\sqrt{35}$

4. Find a unit vector that is orthogonal to both $< -1,2,0 >$ and $< 3,4,-2 >$. $\vec{u} =$ $\left\langle \frac{-4}{\sqrt{120}}, \frac{-2}{\sqrt{120}}, \frac{-10}{\sqrt{120}} \right\rangle$

   OR $\left\langle \frac{4}{\sqrt{120}}, \frac{2}{\sqrt{120}}, \frac{10}{\sqrt{120}} \right\rangle$

Bonus:

1. Let $C = \vec{r}(t)$ and f be as in problem 1. Find formulas for:

   (i) The length of $\vec{r}(t)$ for $a \le t \le b$: $L =$ $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$

   (ii) $\int_C f\, ds =$ $\int_a^b f(x(t), y(t), z(t)) \cdot \sqrt{(x')^2 + (y')^2 + (z')^2}\, dt$

2. Compute the length of $\vec{r}(t) =< \sqrt{7}, \sin^2 t, \cos^2 t >$ for $0 \le t \le \frac{\pi}{4}$:

   Integral Set-up: $\int_0^{\pi/4} \sqrt{2} \sin 2t\, dt$, Answer: $\frac{\sqrt{2}}{2}$