MATH 392 Quiz 2B

June 11, 2019

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Let $\vec{r}(t) = < x(t), y(t), z(t) >$, $f(x,y,z)$ be a scalar function, and $P(x_1, y_1)$ and $Q(x_2, y_2)$ be points in $\mathbb{R}^2$. Complete the following rules/formulas with vector functions (C is a smooth curve parametrized by $\vec{r}(t)$ with $a \le t \le b$, while s is the length of $\vec{r}(t)$. No shorthand, flesh out the full definitions.).

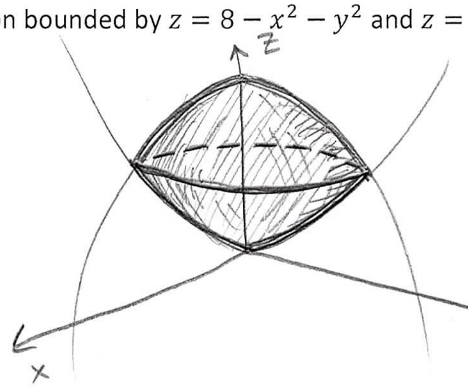
   (a) $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$

   (b) $s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$

   (c) Line segment $\overrightarrow{PQ} = \langle x_1 + (x_2 - x_1)t,\ y_1 + (y_2 - y_1)t \rangle,\ 0 \le t \le 1$ (include limits)

   (d) $\int_C f(x,y,z)\,ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$

2. (a) (2 points) Sketch the region bounded by $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.



   (b) Parametrize the curve of intersection, $\vec{r}_i(t)$, of the above two surfaces. Set up the limits so that the curve is traversed once.

   $\vec{r}_i(t) = \langle 2\cos t, 2\sin t, 4 \rangle$ Limits: $0 \le t \le 2\pi$

3. Setup an integral to find the length of the curve parametrized by $x = 3e^{\theta}\cos\theta$, $y = 3e^{\theta}\sin\theta$ for $0 \le \theta \le \frac{\pi}{4}$.

   $L = \int_0^{\pi/4} 3\sqrt{2}\, e^{\theta}\, d\theta$ (Simplify the integrand, but do not evaluate the integral)

4. Compute $\int_C x\,ds$ where C consists of the quarter circle $x^2 + y^2 = 4$ in the fourth quadrant, traversed clockwise, followed by the line segment from (0,-1) to (0,-2).

   Integral(s) set-up: $\int_{2\pi}^{3\pi/2} 4\cos t\,dt$ Answer: -4

Bonus:

1. Define $\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$