MATH 392 Quiz 2A

June 11, 2019

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Let $\vec{r}(t) =< x(t), y(t) >$, $f(x,y)$ be a scalar function, and $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be points in $\mathbb{R}^3$. Complete the following rules/formulas with vector functions (C is a smooth curve parametrized by $\vec{r}(t)$ with $a \le t \le b$, while s is the arclength of $\vec{r}(t)$. No shorthand, flesh out the full definitions.).

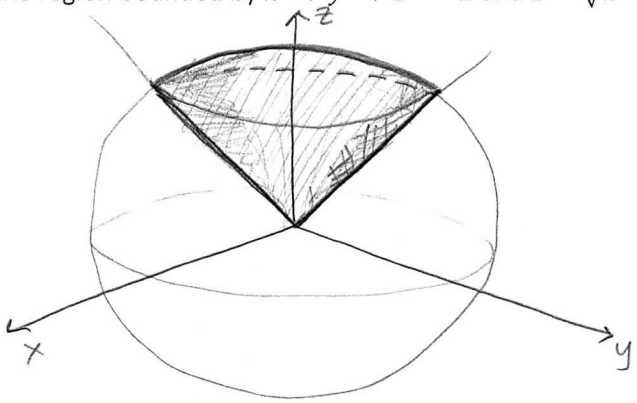
   (a) $\vec{r}\,'(t) =$ $\langle x'(t), y'(t) \rangle$

   (b) Line segment $\overrightarrow{PQ} =$ $\langle x_1+(x_2-x_1)t,\ y_1+(y_2-y_1)t,\ z_1+(z_2-z_1)t \rangle,\ 0 \le t \le 1$ (include limits)

   (c) $s =$ $\int_a^b \sqrt{(x'(t))^2+(y'(t))^2}\,dt$

   (d) $\int_C f(x,y)\,ds =$ $\int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

2. (a) (2 points) Sketch the region bounded by $x^2 + y^2 + z^2 = 2$ and $z = \sqrt{x^2+y^2}$.



   (b) Parametrize the curve of intersection, $\vec{r}_i(t)$, of the above two surfaces. Set up the limits so that the curve is traversed once.

   $\vec{r}_i(t) =$ $\langle \cos t, \sin t, 1 \rangle$ Limits: $0 \le t \le 2\pi$

3. Setup an integral to find the length of the curve parametrized by $x = 2e^\theta \cos\theta$, $y = 2e^\theta \sin\theta$ for $0 \le \theta \le \frac{\pi}{2}$.

   $L =$ $\int_0^{\pi/2} 2\sqrt{2}\, e^\theta\, d\theta$ (Simplify the integrand, but do not evaluate the integral)

4. Compute $\int_C 3y\,ds$ where C consists of the quarter circle $x^2 + y^2 = 1$ in the third quadrant, traversed clockwise, followed by the line segment from (-1,0) to (-2,0).

   Integral(s) set-up: $\int_{3\pi/2}^{\pi} 3\sin t\,dt$ Answer: 3

Bonus:

1. Define $\int_C \vec{F}\cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$