Math 346 Quiz 1
February 6, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. Define a "function" from a set A to a set B: A rule that assigns to each element in A exactly one element in B.

2. Define the "range" of the function above: The set of elements in B that are assigned. (OR, the set of all $y \in B$, such that $f(x) = y$ for some $x \in A$).

3. Let $\mathbb{P}_n$ be the set of all polynomials with real number coefficients of degree n or less. Consider the *function* (really "transformation") $f: \mathbb{P}_3 \to \mathbb{P}_3$ defined by $f(p(x)) = \frac{d}{dx} p(x)$ where $p(x) \in \mathbb{P}_3$.

   (a) Is f one-to-one? NO (b) Is f onto? NO

4. Suppose in problem 3, we had $f: \mathbb{P}_3 \to \mathbb{P}_2$.

   (a) Is f one-to-one? NO (b) Is f onto? YES

5. (2 points) Justify your answers to problem 4.

   ① For any element in $\mathbb{P}_2$, multiple elements from $\mathbb{P}_3$ map to it. For example: all elements of the form $\frac{x^3}{3} + C \longmapsto x^2$. $\Rightarrow$ Not 1-1.

   ② Given $p_1 \in \mathbb{P}_2$, the element $p_2 \in \mathbb{P}_3$ is such that $f(p_2) = p_1$, where p_2 is any anti-derivative of p_1 and $p_2 \in \mathbb{P}_3$.

6. (2 points) **Prove or disprove:** If $n \in \mathbb{Z}$, then $n^2 + n$ is even. (To disprove, give a counter-example—an example that does not fit.) (Many ways to do this! We'll do one).

   Pf: There are two cases. Either (1) n is even or (2) n is odd.

   Case I: If $n = 2k$, $k \in \mathbb{Z}$, then $n^2 + n = (2k)^2 + (2k) = 2(2k^2 + k)$, which is even.

   Case 2: If $n = 2k+1$, $k \in \mathbb{Z}$, then $n^2 + n = (2k+1)^2 + (2k+1) = 2(2k^2 + 3k + 1)$, which is even.

   $\Rightarrow$ In all cases, $n^2 + n$ is even. ∎

Bonus:

1. Given that $A = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 \\ 4 & 7 \end{pmatrix}$, compute the following:

(a) $-2A = \begin{pmatrix} -2 & 0 \\ 2 & -4 \end{pmatrix}$ (b) $3A - 2C = \begin{pmatrix} 1 & -2 \\ -11 & -8 \end{pmatrix}$ (c) $B + C =$ undefined

(d) $AB = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (e) $BC =$ undefined