

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Compute $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ -1 & 2 & -2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = -1$

Write down the first step of your expansion in the first space.

2. Using your answer in problem 1, use Cramer's rule to solve for z in the following system:

$$\begin{aligned} x + z &= 0 \\ 2x + y + 2z &= 4 \\ -x + 2y - 2z &= 9 \end{aligned}$$

Here we have $D_z = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 4 \\ -1 & 2 & 9 \end{vmatrix} = 1 \Rightarrow z = \frac{D_z}{D} = -1$

(Write in the matrix)

Bonus:

1. A system is augmented as $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 2 & 1 & 2 & -7 \\ -1 & 2 & -2 & 21 \end{array} \right)$. How many solutions are there? One!

2. What is the RREF of $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 2 & 1 & 2 & -7 \\ -1 & 2 & -2 & 21 \end{array} \right)$? $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

3. Set $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ -1 & 2 & -2 \end{pmatrix}$. Find $\det(3A^3A^T A^{-1}) = -27$

For problem 1, another good choice is expanding along the second column:

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ -1 & 2 & -2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = -1$$