Math 346 Quiz 4

February 27, 2018

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Name: ANSWERS

**Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.**

1. Solve the following systems, write the answer as a linear combination of column vectors. If no solution exists, write "inconsistent".

(a)
$$\begin{aligned} x - y + 2z &= 3 \\ x + 2y - z &= -3 \\ 2y - 2z &= 1 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 1 & 2 & -1 & -3 \\ 0 & 2 & -2 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 3 & -3 & -6 \\ 0 & 2 & -2 & 1 \end{array}\right) \begin{array}{l} R1 \\ R_2 - R_1 \\ R3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ \boxed{0 \quad 0 \quad 0} & & & \boxed{-5} \end{array}\right) \begin{array}{l} R1 \\ R2/3 \\ 2R2/3 - R3 \end{array}$$

Gasp!

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ Inconsistent!

(b)
$$\begin{aligned} x \quad\quad + z &= 2 \\ 2x + y + 2z &= 4 \\ -x + 2y - 2z &= -3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 2 & 1 & 2 & 4 \\ -1 & 2 & -2 & -3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & -1 \end{array}\right) \begin{array}{l} R_1 \\ R2 - 2R_1 \\ R_1 + R3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \begin{array}{l} R_1 \\ R2 \\ 2R_2 - R3 \end{array}$$

Could stop here, but let's continue.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) R_1 - R3$$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

2. The RREF of a system in variables $(w, x, y, z)$ is $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$. This means: $\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \end{pmatrix} + a\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} -2 \\ 0 \\ -2 \\ 1 \end{pmatrix}$

**Bonus:**

(a) If $A = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$, find $A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$

(b) If $B = \begin{pmatrix} 3 & 3 & 0 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$, find $\det B =$ 6

(c) Is it possible to find $B^{-1}$? How do you know? Yes. $\det B \neq 0$

(d) Let $C$ be the coefficient matrix of the system in problem 1(a). Is it possible to find $C^{-1}$? How do you know? No. RREF of $C \neq I_3$. Also, $\det C = 0$.