Math 346 Quiz 3
February 22, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. True or false: Suppose AB is defined. If A has a row of zeros, then AB has a row of zeros. True

2. Justify your answer in problem 1.
Suppose the i^{th} row of A has all zeros. Then the i^{th} row of AB will have the form $[0 \cdot b_1 \quad 0 \cdot b_2 \ \cdots \ 0 \cdot b_p] = [0 \quad 0 \ \cdots \ 0]$, assuming $A_{n \times m}$, $B_{m \times p}$.
Thus, the i^{th} row of AB will also be all zeros. $\square$

3. Would your answer to problem 1 change if it were B that had the row of zeros? Yes

4. Let $A = [a_{ij}]_{n \times n}$. Define $tr(A) =$ $a_{11} + a_{22} + \cdots + a_{nn}$ OR $\sum_{i=1}^{n} a_{ii}$

5. (a) Given $A = [a_{ij}]_{n \times n}$ and $B = [b_{ij}]_{n \times n}$, conjecture a formula for $tr(A + B) =$ $tr(A) + tr(B)$
(b) Prove your formula works:
Pf: $tr(A+B) = tr[a_{ij} + b_{ij}] = \sum_{i=1}^{n} (a_{ii} + b_{ii}) = \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} b_{ii} = tr(A) + tr(B)$. ∎

6. Solve the system
$$\begin{aligned} x + 2y - z &= 1 \\ x + z &= 3 \\ 2x - 4y + z &= 0 \end{aligned}$$
by: (a) Stating the augmented matrix for the system:
$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 1 & 0 & 1 & 3 \\ 2 & -4 & 1 & 0 \end{array}\right)$$

(b) Find the reduced row-echelon form of the augmented matrix:
$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 1 & 0 & 1 & 3 \\ 2 & -4 & 1 & 0 \end{array}\right)$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 2 & -2 & -2 \\ 0 & 8 & -3 & 2 \end{array}\right) \begin{array}{l} R_2 \\ R_1 - R_2 \\ 2R_1 - R_3 \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -5 & -10 \end{array}\right) \begin{array}{l} R_1 \\ R_2/2 \\ 4R_2 - R_3 \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} R_1 + R_3/5 \\ R_3/-5 + R_2 \\ R_3/-5 \end{array}$$

(c) Write down the solution as a column vector: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

Bonus:

(a) Justify your answer to problem 3.
Counter-example: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \Rightarrow AB = \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}$. So B has a row of zeros, but AB does not. (There are many such examples).

(b) Recall: if $A = [a_{ij}]_{n \times m}$, then $A^T = [a_{ji}]_{m \times n}$. Suppose $B = [b_{ij}]_{n \times m}$. Prove that $(A + B)^T = A^T + B^T$.
Pf: $(A+B)^T = [a_{ij} + b_{ij}]^T = [a_{ji} + b_{ji}] = [a_{ji}] + [b_{ji}] = A^T + B^T$ ∎